Monitoring for Power-line Change and Outage Detection in Smart Grid via the Alternating Direction Method of Multipliers

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***Abstract*—A novel distributed line outage detection algorithm was developed in this paper through convex relaxation and alternating direction method of multipliers (ADMM) method for smart grid system. The devised approach allows identification of multiple line outages using limited number of PMU measurements. The diagnosis procedure is performed close to the place where PMU measurements are collected and only partial variable estimates are exchanged among the neighbours of processors. It is shown that the proposed method outperforms the existing methods, which are either suffering from computational complexity or security and privacy issues. Numerical tests demonstrated the merits of the proposed schemes in coordinately figuring out multiple line outages in the system.**

***Keywords*—Line outage detection; Convex optimization; Smart Grid; Distributed computing; Alternating direction method of multipliers.**

I. Introduction

Recently the topic of Cyber-Physical Systems (CPS) has been attracting much attention in computer science society.

CPS are integrations of computation, networking, and control for physical processes in which the physical system can affect the cyber system and vice versa. Specifically, power grid is seen as one of such CPS and form a rich environment for the study of several inherent problems. In the first place, it becomes one of the largest and most complex interconnected networks in the world and the corresponding control task is extremely difficult due to its vast scale. Second, new kind of power transfers resulting from use of distributed energy generation and storage will potentially make power systems increasingly vulnerable to cascading failures in which a small series of events can lead to a major blackout [1]. Third, the new types of security issues arise in CPS, which is a growing concern in smart grid systems. For instance, a fault or malicious attack in the communication network can mislead the decisions in centralized control center that will cause dramatically changes in the physical power grid, such as line overflows, generator outages, etc. Thus, advanced smart grid system calls for distributed computation, communication and control framework that local actions can be coordinated for integrated and efficient protection of the power grid as a whole.

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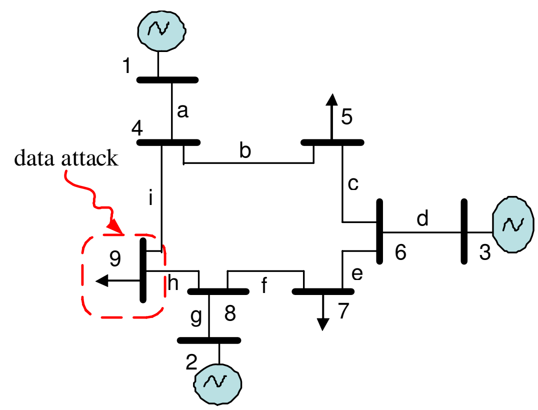

Fig. 1. IEEE 9-bus system with data attack on bus No. 9

To demonstrate the validity of the security concerns above, We test the IEEE 9-bus power system in Fig. 1 using the MATPOWER simulation package under the default settings specified by the IEEE document, and we obtain a set of solutions which include the amount of power flow at each branch and the voltage magnitude at each bus [11]. Suppose the message sent from bus No. 9 to the control center (i.e., SCADA) containing the load information is intercepted and tampered by malicious attackers, which induces a false change of the demanded power at bus No. 9 from the original 125 MW to 560 MW while the overall power demand of this system remains under its maximum generation capacity. We retest this 9-bus system and we find that as a result, significant power overflows will happen to branch a, b, and i, and the voltage magnitudes at bus $4-9$ will violate the prescribed limit remarkably. The stability of this power system is thus in great jeopardy due to this malicious data attack.

The rest of the paper is organized as following. Section II presents the the related works of line outage detection in smart grid. The system model, problem formulation and related preliminary are discussed in section III. The design of distributed algorithms are presented in section IV. In section V, we analyze and discuss the simulation results. We then conclude the paper in section VII.

II. Related Work

Nowadays, the advances in information infrastructure provide opportunities to better cope with the reliability related issues in smart grid. The development of real-time synchronized phasor measurement units (PMUs) is enabling direct usage of PMU-provided measurements to detect events within the power system, which makes PMU-based line outage detection a promising direction to help improve the chances of fault identification.

Existing PMU-based line outage detection methods typically use the internal-external network model for the whole interconnected system in which the goal is to identify external line outages using only measurements within the internal system [4], [9], [10], [12]. [9] formulates the line outage detection as a best match problem which contains an exhaustive searching process for the most likely outaged line. Thus it can only handle single line outage scenario. Building upon the work [9], double line outage detection is considered in [10] while it restricts to the case with exactly double line outaged in the system. A similar exhaustive search is also applied in [10] but the searching space is even much larger than single line's, which is very computational expensive. Another method to line-outage identification employs Gauss-Markov graphical model of the power network and is capable of dealing with multiple outages at moderate complexity [7] while it needs the measurements across the grid. An alternative sparse overcomplete representation based algorithm was proposed in [12], which can also handle multiple line outages. However, as aforementioned methods, they are all carrying out the processing in a centralized manner, which is vulnerable in practice. Further, these existing approaches need to transmit raw data in the system that may encounter privacy issues. The objective of this paper is to design a distributed line outage detection scheme for smart grid system upon the network of Wide Area Measurement System (WAMS). An example of WAMS is showed in Fig. 4.

III. Approach Procedure Description and Problem Formulation

Our detection approach is based on the network of WAMS. In each area of WAMS, certain number of PMUs are implemented in the buses. They will measure the bus voltage phasor and all the branch-current phasors that incident to the corresponding buses. In the higher level, there is a Phasor Data Concentrator (PDC) collecting the data from PMUs within its area. The proposed line outage detection is then performed among the PDCs in a distributed fashion. At last, the results after detection not the raw data are transmitted to the WAMS

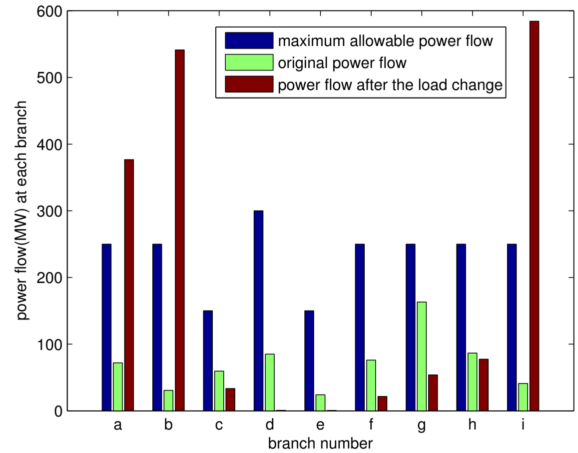

Fig. 2. Power flow at each branch before and after attack.

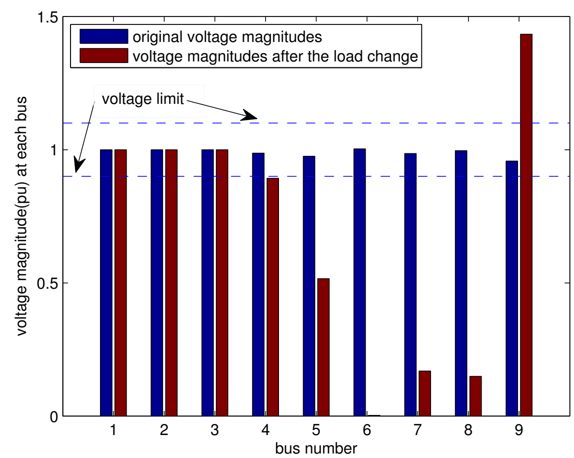

Fig. 3. Voltage magnitude at each branch before and after attack.

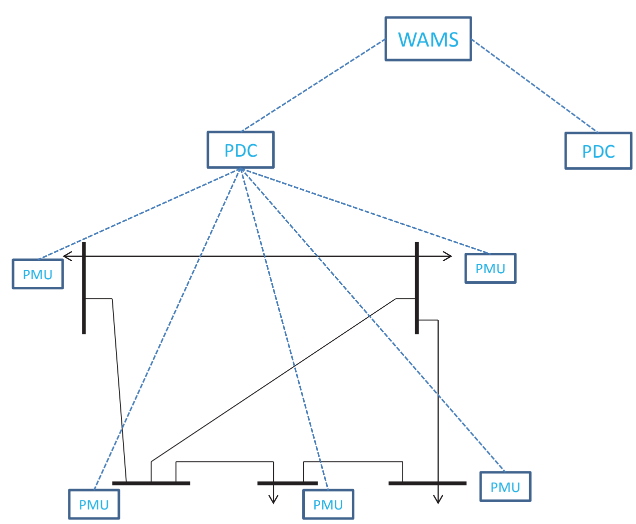

Fig. 4. Architecture of WAMS

center aiming at providing critical information to the system operators.

A. System Model

In typical power transmission system, the synchrophasor measurements at the $\mathbf{n}$-th PDC area expressed in rectangular coordinates are collected in a vector $\bar{\mathbf{y}}_\mathbf{n}$ and obey the following linear model:

$$\bar{\mathbf{y}}_\mathbf{n} = \bar{\mathbf{H}}_\mathbf{n}\mathbf{x} + \bar{\mathbf{g}}_\mathbf{n} \tag{1}$$

where $\mathbf{x}$ is the state of the whole system containing all line currents. $\bar{\mathbf{H}}_\mathbf{n} \in \mathbb{R}^{M_n \times 2N_l}$ is the measurement matrix, M_n is the number of measurements within $\mathbf{n}$-th PDC area, N_l is the number of lines in the whole system. $\bar{\mathbf{g}}_\mathbf{n} \sim \mathcal{N}(\mathbf{0}, \mathbf{\Lambda}_\mathbf{n})$ denotes the additive Gaussian noise vector. For notational convenience, we multiply with $\mathbf{\Lambda}_\mathbf{n}^{-1/2}$ on both sides of (1) to yield:

$$\mathbf{y}_\mathbf{n} = \mathbf{H}_\mathbf{n}\mathbf{x} + \mathbf{g}_\mathbf{n} \tag{2}$$

where $\mathbf{y}_\mathbf{n} = \mathbf{\Lambda}_\mathbf{n}^{-1/2}\bar{\mathbf{y}}_\mathbf{n}$, and the other terms are manipulated similarly.

Let $\mathbf{v} = Re(\mathbf{v}) + Im(\mathbf{v})$ be the $N_b \times 1$ vector of complex nodal voltages with N_b the number of buses in the system. By writing down the node equations of Kirchhoff's current law (KCL) and Kirchhoff's voltage law (KVL) at each node, we can derive the vector of complex currents injected on each line as follows:

$$\mathbf{i}_\mathbf{fl} = \mathbf{x} = \mathbf{Y}_\mathbf{fl}\mathbf{v} \tag{3}$$

where $\mathbf{Y}_\mathbf{fl}$ describes the line-to-bus admittance matrix. The matrices $\bar{\mathbf{H}}_\mathbf{n}$ in (1) can be expressed as:

$$\bar{\mathbf{H}}_\mathbf{n} = \begin{pmatrix} \mathbf{Q}_\mathbf{n}Re(\mathbf{Y}_\mathbf{fl}^{-1}) & -\mathbf{Q}_\mathbf{n}Im(\mathbf{Y}_\mathbf{fl}^{-1}) \\ \mathbf{Q}_\mathbf{n}Im(\mathbf{Y}_\mathbf{fl}^{-1}) & \mathbf{Q}_\mathbf{n}Re(\mathbf{Y}_\mathbf{fl}^{-1}) \\ \mathbf{e}_\mathbf{n}^\mathbf{T} & \mathbf{0}^\mathbf{T} \\ \mathbf{0}^\mathbf{T} & \mathbf{e}_\mathbf{n}^\mathbf{T} \end{pmatrix} \tag{4}$$

where $\mathbf{Q}_\mathbf{n}$ is the selection matrix according to the $\mathbf{n}$-th PDC.

B. Possible Centralized Solution for Line Outage Detection

In this paper, we combine the measurements and the prior information on the state to do line outage detection. We denote $\tilde{\mathbf{x}}$ as the statistics of historical data on the transmission line currents, which follows normal distribution with mean vector $\mathbf{x}_\mathbf{p}$ and covariance matrix $\mathbf{\Lambda}_\mathbf{p}$, that is $\tilde{\mathbf{x}} \sim \mathcal{N}(\mathbf{x}_\mathbf{p}, \mathbf{\Lambda}_\mathbf{p})$. We assume the state variables are independent which implies the covariance matrix $\mathbf{\Lambda}_\mathbf{p}$ is diagonal. Inspired by the idea of compressive sensing, we can have sparse solution for certain underdetermined system by adding the ℓ_1-norm regularization [3]. Since most of the components of the item in ℓ_1-norm term is pushed into zero, we let the unknown state vector $\mathbf{x}$ to compare with its nominal model in the ℓ_1-norm term in order to create "sparse" faulty branches.

Now we suppose the line outage detection is performed in a *single* control center. We employ the ℓ_1-norm approximation in [3] leading to a convex optimization criterion below:

$$\min_{\mathbf{x}} \ \frac{1}{2}\left\|\mathbf{y} - \mathbf{H}\mathbf{x}\right\|_2^2 + \lambda\left\|\mathbf{\Lambda}_\mathbf{p}^{-1/2}(\mathbf{x} - \mathbf{x}_\mathbf{p})\right\|_1 \tag{5}$$

Note that if we decompose (5) into N PDC areas, then (5) can be expressed in the following:

$$\min_{\mathbf{x}_\mathbf{n}} \sum_{\mathbf{n}=1}^{N} f_\mathbf{n}(\mathbf{x}_\mathbf{n}) \tag{6}$$

accordingly the "cost function" for each PDC is:

$$f_\mathbf{n}(\mathbf{x}_\mathbf{n}) = \frac{1}{2}\|\mathbf{y}_\mathbf{n} - \mathbf{H}_\mathbf{n}\mathbf{x}_\mathbf{n}\|_2^2 + \lambda\left\|\mathbf{\Lambda}_\mathbf{pn}^{-1/2}(\mathbf{x}_\mathbf{n} - \mathbf{x}_\mathbf{pn})\right\|_1 \tag{7}$$

where $\mathbf{x}_\mathbf{n}$, $\mathbf{H}_\mathbf{n}$, $\mathbf{x}_\mathbf{pn}$ and $\mathbf{\Lambda}_\mathbf{pn}$ are corresponding to the states involved in $\mathbf{n}$-th PDC. Each PDC in the area can choose to minimize (7) individually but this method is clearly sub-optimal since the overlapping states are not taken into account.

IV. Proposed Distributed Line Outage Detection

In this section, we derive to solve the optimization problem in (6) in a distributed manner. Denote $\mathbf{x}_\mathbf{n}$ as the subset of $\mathbf{x}$, which contains the states involved in $\mathbf{n}$-th PDC. Also denote $\mathbf{x}_\mathbf{nm}$ as the value of the sharing states between neighbouring $\mathbf{n}$-th and $\mathbf{m}$-th PDC (a sub-vector of $\mathbf{x}_\mathbf{n}$ or $\mathbf{x}_\mathbf{m}$). Then the estimate of overlapping state variables by neighbouring PDCs should be the same. Then equation (6) can be reformulated as:

$$\begin{aligned} \underset{\mathbf{x}_\mathbf{n}}{\text{minimize}} \quad & \sum_{\mathbf{n}=1}^{N} f_\mathbf{n}(\mathbf{x}_\mathbf{n}) \\ \text{subject to} \quad & \mathbf{x}_\mathbf{nm} = \mathbf{x}_\mathbf{mn}, \ \mathbf{m} \in \mathcal{N}_\mathbf{n}; \ \mathbf{n}, \mathbf{m} \in P \end{aligned} \tag{8}$$

where $\mathcal{N}_\mathbf{n}$ is the set of neighbouring PDCs of $\mathbf{n}$-th PDC, P is the set of PDCs.

Let us now apply the Alternating Direction Method of Multipliers (ADMM) [2] to solve the line outage detection problem formulated in (8) using a distributed mechanism. We introduce auxiliary variables $\vartheta_\mathbf{nm}$ and $\mathbf{z}_\mathbf{n}$ in order to fit the ADMM framework. Then, (8) can be alternatively expressed as:

$$\begin{aligned} \underset{\mathbf{x}_\mathbf{n}, \vartheta_\mathbf{nm}, \mathbf{z}_\mathbf{n}}{\text{minimize}} \quad & \sum_{\mathbf{n}=1}^{N} f_\mathbf{n}(\mathbf{x}_\mathbf{n}) \\ \text{subject to} \quad & \mathbf{x}_\mathbf{nm} = \vartheta_\mathbf{nm}, \ \mathbf{m} \in \mathcal{N}_\mathbf{n}; \ \mathbf{n}, \mathbf{m} \in P \\ & \mathbf{x}_\mathbf{n} - \mathbf{x}_\mathbf{pn} = \mathbf{z}_\mathbf{n} \end{aligned} \tag{9}$$

We also introduce variable $\nu_\mathbf{nm}$ denoting the lagrangian multiplier for each constraint in first constraint in (9). $\mathbf{s}_\mathbf{n}$ denotes the multiplier for the second constraint in (9). Note that by using ADMM in our problem, there are three primal variables: $\mathbf{x}_\mathbf{n}$, $\vartheta_\mathbf{nm}$ and $\mathbf{z}_\mathbf{n}$; two dual variable: $\nu_\mathbf{nm}$ and $\mathbf{s}_\mathbf{n}$. Firstly the augmented Lagrangian function can be obtained as:

$$\begin{aligned} & L_\rho(\mathbf{x}_\mathbf{n}, \vartheta_\mathbf{nm}, \mathbf{z}_\mathbf{n}, \nu_\mathbf{nm}, \mathbf{s}_\mathbf{n}) \\ & = \sum_{\mathbf{n}=1}^{N} \Big\{ f_\mathbf{n}(\mathbf{x}_\mathbf{n}) + \sum_{\mathbf{m} \in \mathcal{N}_\mathbf{n}} (\nu_\mathbf{nm}^T(\mathbf{x}_\mathbf{nm} - \vartheta_\mathbf{nm}) \\ & + (\rho/2)\left\|\mathbf{x}_\mathbf{nm} - \vartheta_\mathbf{nm}\right\|_2^2) + \mathbf{s}_\mathbf{n}^T(\mathbf{x}_\mathbf{n} - \mathbf{x}_\mathbf{pn} - \mathbf{z}_\mathbf{n}) \\ & + (\rho/2)\left\|\mathbf{x}_\mathbf{n} - \mathbf{x}_\mathbf{pn} - \mathbf{z}_\mathbf{n}\right\|_2^2 \Big\} \end{aligned} \tag{10}$$

where ρ is a predefined constant. Let k to be the iteration index, then ADMM algorithm consists of the following update rules:

$$\mathbf{x_n}^{k+1} = \arg\min_{\mathbf{x_n}} \ L_\rho(\mathbf{x_n}, \vartheta_{\mathbf{nm}}^k, \mathbf{z_n}^k, \nu_{\mathbf{nm}}^k, \mathbf{s_n}^k) \quad (11a)$$

$$(\vartheta_{\mathbf{nm}}^{k+1}, \mathbf{z_n}^{k+1}) = \arg\min_{\vartheta_{\mathbf{nm}}, \mathbf{z_n}} \ L_\rho(\mathbf{x_n}^{k+1}, \vartheta_{\mathbf{nm}}, \mathbf{z_n}, \nu_{\mathbf{nm}}^k, \mathbf{s_n}^k) \quad (11b)$$

$$\nu_{\mathbf{nm}}^{k+1} = \nu_{\mathbf{nm}}^k + \rho(\mathbf{x_{nm}}^{k+1} - \vartheta_{\mathbf{nm}}^{k+1}) \quad \text{for all } \mathbf{n}, \mathbf{m}. \quad (11c)$$

$$\mathbf{s_n}^{k+1} = \mathbf{s_n}^k + \rho(\mathbf{x_n}^{k+1} - \mathbf{x_{pn}} - \mathbf{z_n}^{k+1}) \quad (11d)$$

Now, we are considering how to implement the updates in (11a)-(11d) efficiently. Since (11c) and (11d) are simple linear updating equations, we only need to focus on deduction of (11a) and (11b). To solve (11a), we use some tricks to enable simplification of the analysis. We define:

1) $\mathbf{D_n}$ as a diagonal matrix with its $(\mathbf{m}, \mathbf{m})$-th entry to be 1;
2) $\mathbf{r_n}^k = \vartheta_{\mathbf{n}}^k - (1/\rho)\nu_{\mathbf{n}}^k$;
3) $\mathbf{I_n}$ denotes an identity matrix with its dimension to be the number of states in $\mathbf{n}$-th area.

As a result, the term $\sum_{\mathbf{m} \in \mathcal{N}_\mathbf{n}} (\frac{\rho}{2}) \left\| \mathbf{x_{nm}} - \vartheta_{\mathbf{nm}} + (\frac{1}{\rho})\nu_{\mathbf{nm}}^k \right\|_2^2$ in (11a) can be expressed as: $(\rho/2) \left\| \mathbf{D_n}(\mathbf{x_n} - \mathbf{r_n}^k) \right\|_2^2$. Then after manipulating using matrix calculus, we obtain the minimizer of (11a) as following:

$$\begin{aligned} \mathbf{x_n}^{k+1} &= (\mathbf{H_n}^T \mathbf{H_n} + \rho \mathbf{D_n} + \rho \mathbf{I_n})^{-1} \\ &\times \left(\mathbf{H_n}^T \mathbf{y_n} + \rho(\mathbf{D_n} \mathbf{r_n}^k + \mathbf{x_{pn}} + \mathbf{z_n}^k - (1/\rho)\mathbf{s_n}^k) \right) \end{aligned} \quad (12)$$

For solving (11b), it is known that the optimality conditions satisfy when zero vector belongs to subdifferentials of (11b) with respect to variable $\vartheta_{\mathbf{nm}}$ and $\mathbf{z_n}$ [8]. We first consider the minimization with $\vartheta_{\mathbf{nm}}$. An important fact can be observed that will help infer the updates of $\vartheta_{\mathbf{nm}}$: For each pair of $\mathbf{n}, \mathbf{m}$ in (11c), $\nu_{\mathbf{nm}}^k + \nu_{\mathbf{mn}}^k = 0$.

At this point, it is clear to see that by using the fact above, we can derive that:

$$\vartheta_{\mathbf{nm}}^{k+1} = (\mathbf{x_{nm}}^{k+1} + \mathbf{x_{mn}}^{k+1})/2 \quad (13)$$

Next, we are concerning how to approach the updates of $\mathbf{z_n}$. Note that due to the ℓ_1-norm term, (11b) is not differentiable everywhere but subdifferentiable with respect to $\mathbf{z_n}$ [8]. As mentioned in previous, we take the subdifferential over (11b) with respect to $\mathbf{z_n}$ and the optimality condition becomes:

$$\mathbf{0} \in \partial \lambda \left\| \mathbf{\Lambda_{pn}}^{-1/2} \mathbf{z_n} \right\|_1 + \rho\big(\mathbf{z_n} - (\mathbf{x_n}^{k+1} - \mathbf{x_{pn}} + (1/\rho)\mathbf{s_n}^k)\big)$$

By using the soft thresholding operator defined in [2], for instance, the i-th component $\mathbf{z_n}^{k+1}[i]$ (scalar) is updated as:

$$\mathbf{z_n}^{k+1}[i] = S_{(\lambda/\rho)\mathbf{\Lambda_{pn}}^{-1/2}[i][i]}(\mathbf{x_n}^{k+1}[i] - \mathbf{x_{pn}}[i] + (1/\rho)\mathbf{s_n}^k[i])$$

In a similar way, a closed-form solution for the updates of $\mathbf{z_n}$ is obtained as follows:

$$\mathbf{z_n}^{k+1} = S_{(\lambda/\rho)\mathbf{\Lambda_{pn}}^{-1/2}}(\mathbf{x_n}^{k+1} - \mathbf{x_{pn}} + (1/\rho)\mathbf{s_n}^k) \quad (14)$$

where $S_b(a) = (a - b)_+ - (-a - b)_+$. Note here component-wise updating is applied that i-th component of $\mathbf{z_n}$ is updated according to the i-th entry of the rest of the vectors in (14) and (i, i)-th entry of the diagonal matrix $\mathbf{\Lambda_{pn}}^{-1/2}$.

Now the ADMM updating in (11a)-(11d) for each processor can be summarized in Algorithm 1.

Algorithm 1 Distributed Line Change Detection (D-LCD)

Input: $\mathbf{y_n}, \mathbf{H_n}, \mathbf{\Lambda_n}, \mathbf{\Lambda_{pn}}, \mathbf{x_{pn}}, \mathbf{D_n}, \lambda > 0, \rho > 0, k = 0.$
Initialize: $\mathbf{x_n}, \vartheta_{\mathbf{nm}}, \mathbf{z_n}, \nu_{\mathbf{nm}}, \mathbf{s_n}$.
while *stopping criterion not reached* **do**
  $k \leftarrow k + 1$.
  Update $\mathbf{x_n}^{k+1}$ based on (12).
  Exchange $\mathbf{x_{nm}}^{k+1}$ with its neighbours.
  Update $\vartheta_{\mathbf{nm}}^{k+1}$, $\mathbf{z_n}^{k+1}$ via (13) and (14) respectively.
  Update $\nu_{\mathbf{nm}}^{k+1}$ and $\mathbf{s_n}^{k+1}$ through (11c) and (11d).
end while

V. Preliminary Numerical Results

For evaluating the proposed centralized and distributed line change detection algorithms, we use the Intel Duo Core @1.8 GHz (1.5GB RAM) computer with MATLAB for numerical testing. The state variables and measurements are obtained from MATPOWER [13]. To solve the centralized algorithm in (5), we used CVX, a package for specifying and solving convex programs [6], [5]. The PMU measurement noise is simulated as independent zero-mean Gaussian with its covariance matrix $\mathbf{\Lambda_n} = 0.002\mathbf{I_n}$. The covariance matrix of the prior state vector is considered to be $\mathbf{\Lambda_p} = 0.003\mathbf{I_p}$, where $\mathbf{I_p}$ is an identity matrix with the same dimension as the state vector.

A. WSCC 9-Bus Test Case

In this section, the WSCC 9-Bus Test Case System was used for our simulation. The diagram of the system is demonstrated in Fig. 5. There are three generators (G1,G2,G3), three transformers (T1,T2,T3).

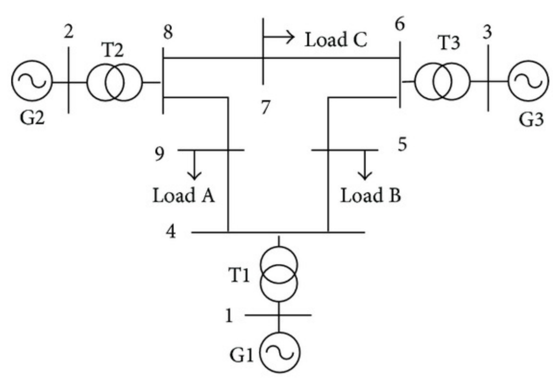

Fig. 5. WSCC 9-Bus Test Case System

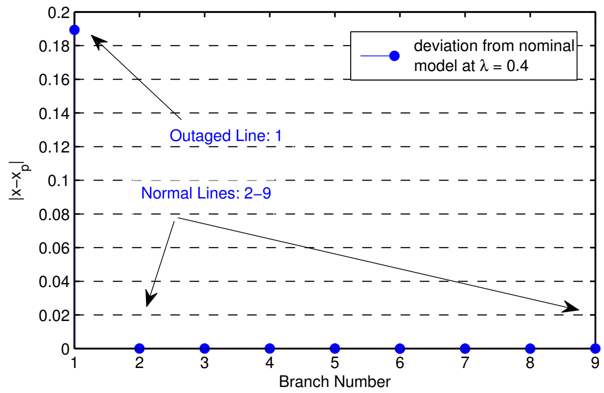

Fig. 6. Centralized Line Change Detection

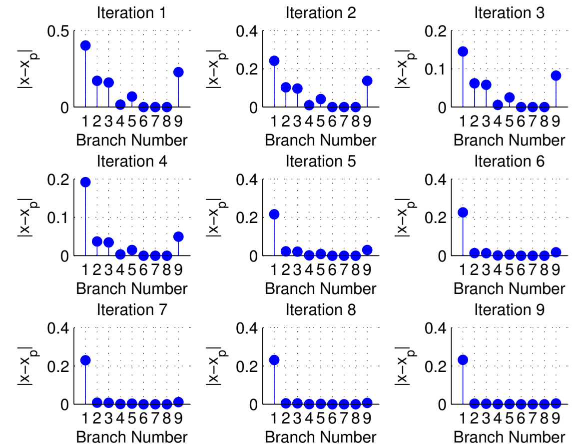

Fig. 7. Distributed Line Change Detection

From line parameters of the system, the line-to-bus admittance matrix $\mathbf{Y_{fl}}$ can be formed which is used for constructing the measurement matrix $\mathbf{H}$ in (5). The system is assumed to be at steady state before and after the line change. We made the line change on the reactance of line $(1-4)$ which was altered from 0.0576 to Infinity. Then we ran DC power flow in MATPOWER to obtain the state vector in normal condition and the measurements after change. The above are all the quantities considered as the input to our centralized line change detection algorithm. The result in Fig. 6 shows that the faulty line $(1-4)$ has been correctly detected by the algorithm.

We also tested our D-LCD algorithm on this 9-Bus system and the results of first nine ADMM iterations are captured in Fig. 7. Note that initially branch 1,2,3,5,9 have positive values which means they are all seen as a group of possible faulty lines. During iteration 2-4, the values of branch 1,2,3,5,9 are actually decreasing while an interesting point is that the decreasing speed of branch 2,3,5,9 is much faster than branch 1's. This observation is conformed with the theory part discussed in previous that the most likely set of branches should survive for the next iteration.

VI. Conclusion

The vulnerability of smart grid system was demonstrated which implies the urgent demand of advanced and distributed fault detection methods. We utilized ADMM-based method to solve the line outage detection problem in a distributed fashion. The proposed method is capable of detecting multiple line outages using limited PMU measurements. It was illustrated via numerical examples that the devised novel approach can coordinately figure out line outages in the smart grid system. Studying the proposed scheme for practical power network dimensions is currently under investigation.

References

[1] R. Baldick, B. Chowdhury, I. Dobson, Z. Dong, B. Gou, D. Hawkins, Z. Huang, M. Joung, J. Kim, D. Kirschen, S. Lee, F. Li, J. Li, Z. Li, C.-C. Liu, X. Luo, L. Mili, S. Miller, M. Nakayama, M. Papic, R. Podmore, J. Rossmaier, K. Schneider, H. Sun, K. Sun, D. Wang, Z. Wu, L. Yao, P. Zhang, W. Zhang, and X. Zhang, "Vulnerability assessment for cascading failures in electric power systems," in *Power Systems Conference and Exposition, 2009. PSCE '09. IEEE/PES*, 2009, pp. 1–9.

[2] S. Boyd, N. Parikh, E. Chu, B. Peleato, and J. Eckstein, "Distributed optimization and statistical learning via the alternating direction method of multipliers," *Found. Trends Mach. Learn.*, vol. 3, no. 1, pp. 1–122, Jan. 2011. [Online]. Available: http://dx.doi.org/10.1561/2200000016

[3] D. Donoho, "Compressed sensing," *IEEE Transactions on Information Theory*, vol. 52, no. 4, pp. 1289–1306, 2006.

[4] R. Emami and A. Abur, "Tracking changes in the external network model," in *North American Power Symposium (NAPS)*, 2010, pp. 1–6.

[5] M. Grant and S. Boyd, "Graph implementations for nonsmooth convex programs," in *Recent Advances in Learning and Control*, ser. Lecture Notes in Control and Information Sciences, V. Blondel, S. Boyd, and H. Kimura, Eds. Springer-Verlag Limited, 2008, pp. 95–110, http://stanford.edu/~boyd/graph_dcp.html.

[6] ——, "CVX: Matlab software for disciplined convex programming, version 2.0 beta," http://cvxr.com/cvx, Sep. 2013.

[7] M. He and J. Zhang, "Fault detection and localization in smart grid: A probabilistic dependence graph approach," in *Smart Grid Communications (SmartGridComm), 2010 First IEEE International Conference on*, 2010, pp. 43–48.

[8] R. Rockafellar, *Convex Analysis*, ser. Princeton landmarks in mathematics and physics. Princeton University Press, 1997. [Online]. Available: http://books.google.com/books?id=1TiOka9bx3sC

[9] J. Tate and T. Overbye, "Line outage detection using phasor angle measurements," *IEEE Transactions on Power Systems*, vol. 23, no. 4, pp. 1644–1652, 2008.

[10] ——, "Double line outage detection using phasor angle measurements," *IEEE Power Energy Society General Meeting*, pp. 1–5, 2009.

[11] L. Zhao and R. Blum, "Sampling rate impact on intrusion and fault detection in smart grid with new locally optimum tests," in *Integrated Networks for Electricity Workshop: Toward the Smart Grid*, January 20th, 2012.

[12] H. Zhu and G. Giannakis, "Sparse overcomplete representations for efficient identification of power line outages," *IEEE Transactions on Power Systems*, vol. 27, no. 4, pp. 2215–2224, 2012.

[13] R. Zimmerman, C. Murillo-Sanchez, and R. Thomas, "Matpower: Steady-state operations, planning, and analysis tools for power systems research and education," *IEEE Transactions on Power Systems*, vol. 26, no. 1, pp. 12–19, 2011.