

Distributed Topology Control for Efficient OSPF Routing in Multi-hop Wireless Networks

Xiaogang Yang[†] Xufei Mao[‡] Guangyu Pei^{*} Wen-Zhan Song[†]

[†] Sensorweb Research Laboratory, Washington State University.

[‡] Department of Computer Science, Illinois Institute of Technology.

^{*} Boeing Phantom Works.

Email: xiaogang_yang@wsu.edu, xmao3@iit.edu, Guangyu.pei@boeing.com, songwz@wsu.edu

Abstract—This paper studies distributed topology control algorithms to support the efficient Open Shortest Path First (OSPF) link state routing in multi-hop wireless networks. It is highly desirable to retain the basic OSPF model of reliable flooding, especially when large quantities of external, rarely-changing routing data must be carried across the radio network. However, it is not easy to implement reliable flooding, and improper implementations may cause excessive message control overheads. An existing patent method proposes an expanding ring algorithm to build adjacency graph for reliable flooding. We first analyze this method and show the message complexity of reliable flooding could be $O(n)$ times of optimum. We then show that a slight modification on this method can generate an adjacency graph, called ER-CDS, which supports reliable flooding with message complexity $O(1)$ of optimum. However, the construction cost based on the patent method is still high and does not adapt to network topology changes well. Finally, we propose a localized splash merging algorithm to construct ER-CDS, and then conducted extensive simulations to evaluate its performance.

keyword: Topology control, Connected Dominating Set, Reliable Flooding

I. INTRODUCTION

Existing wired communication networks use various algorithms for disseminating routing data necessary for routing packets from a source node to a destination node. Each node of the network that handles packets needs sufficient knowledge of the network topology such that it can choose the right output interface through which to forward received packets. Link state routing algorithms, such as Open Shortest Path First (OSPF) algorithm, permit the construction of a network topology such that any given node in the network may make packet-forwarding decisions.

However, a number of difficulties may arise if OSPF is implemented over a multi-hop, multi-access packet radio network with its own private, internal routing system. Although the routing system may enable it to appear to be much like a wired network, the properties and characteristics are nevertheless much different from those of wired point-to-point or multi-access networks. Likewise, the adoption of the standard OSPF point-to-multipoint network model for distribution of routing information over a multi-hop routing one is equally inappropriate. The links employed by a point-to-multipoint model to represent the radio networks are likely

to be several radio hops in length, and so their use to distribute routing information would often require packet replication and/or result in transmitting duplicate information over a single radio link. Furthermore, the network of links needed by a point-to-multipoint model to represent the radio network may be much more dense than required for distribution of routing information. Finally, this network of links may be constantly changing in response to the need for accurate representation of the radio network, and so may not be sufficiently stable for effective use in distributing routing information.

On the other hand, it is highly desirable to retain the basic OSPF model of reliable flooding, especially when large quantities of external, rarely-changing routing data must be carried across the radio network. Therefore, there exists a need for systems and methods that can solve some of the inherent problems that exist with distributing OSPF routing information across a multi-hop, multi-access packet radio network, while maintaining full compatibility with standard OSPF over the networks and preserving the basic OSPF model of reliable flooding.

An existing patent [1] proposes an expanding ring algorithm to build adjacency graph for reliable flooding. We first analyze this method and show the message complexity of reliable flooding based on it could be $O(n)$ times of optimum where n is the network size. We then show that a slight modification on it can generate a adjacency graph, called ER-CDS, which supports reliable flooding with message complexity $O(1)$ of optimum. However, the construction cost based on the patent method is still high and does not adapt to network topology changes well. Finally, we propose a localized splash merging algorithm to construct ER-CDS, and extensive simulation results show that our ER-CDS has better performance than others in most aspects. ER-CDS stands for Expanding Ring based Connected Dominating Set. In other words, our proposed splash merging algorithm will form a Connected Dominating Set (CDS) to support reliable flooding of OSPF routing information. Basically all nodes will be divided into three groups, *dominators*, *connectors* and *dominatees*, and all dominators and connectors will form the CDS and can be considered as the virtual backbone (infrastructure). To disseminate the OSPF routing information, the link state packets originated from a dominatee will first be forwarded to its dominator, which

then broadcasts to all its dominatees and adjacent dominators in CDS (through connectors), until all of them confirm the reception. Every adjacent dominator will repeat this process until reliable flooding to the whole network is accomplished. In other words, only dominators and connectors will participate in message re-broadcasting, which significantly reduce the message overheads of flooding. In literature, the application of CDSs in the wireless network have been widely studied. For example, [4], [5] formed an underlying architecture used by protocols which included media access coordination; Datta and Sto in [6] used CDS to do location-based routing; the issues related to energy conservation have been studied in [7]–[9]; and Budhaditya *et al.*, and Ding *et al.*, studied how to use CDS to do topology control in [10] and [11] respectively. Actually, many works seek a Minimum Connected Dominating Set (MCDS) in unit-disk graphs as their major design goals. This is because the system control-related messages from the base station or sink node to all wireless nodes, will count on the virtual backbone (nodes on the CDS). Thus performance is directly related to the properties of the CDS. Intuitively, minimizing the cardinality of the computed CDS can help to decrease the control overhead for most of the application, such as broadcasting for route discovery in [12], [13]. However, constructing MCDS is a well-known NP-hard problem. In this paper, we will show that the ER-CDS constructed by our splash merging algorithm has the following properties: (1) The CDS size is bounded by $12|MCDS|$. (2) The time and message complexity of splash merging algorithm is $O(|DS|)$ and $O(n + |DS||CDS|)$ where $|DS|$ and $|CDS|$ is the cardinality of DS and CDS of ER-CDS.

The rest of the paper is organized as follows. In section II, we analyze an existing patent method and propose an improved structure, called ER-CDS, to guarantee worst-case performance. In III, we propose an innovative splash merging algorithm to construct ER-CDS, with low message complexity and high adaptivity to topology changes. We conducted extensive simulations on TOSSIM 2.0 and compared our algorithms to other CDS algorithms. Simulation results are shown in section IV. Related works have been presented in section V. Finally, we conclude the work in section VI.

II. EXPANDING RING GRAPH ANALYSIS

A multi-hop wireless *ad hoc* network is modeled by a set V of n wireless nodes distributed in a two-dimensional plane. Each node has the same *maximum* transmission range. By proper scaling, we assume that all nodes have the maximum transmission range equal to one unit. These wireless nodes define a *unit disk graph* $UDG(V)$ in which there is an edge between two nodes iff the Euclidean distance between them is at most one unit. In other words, we assume that two nodes can always receive the signal from each other directly if the Euclidean distance between them is no more than one unit. Notice that, in practice, the transmission region of a node is not necessarily a perfect *disk*. As most experiment results found in literature, for simplicity, we model it by *disk* in order to first explore the underlying nature of ad hoc networks. It has

been proved in [30] that for a wireless network with n nodes falling into a unit square, when $\pi nr^2 \geq \log n$, the network is connected with high probability where r is the transmission range of the ordinal wireless nodes. Clearly, by a proper scaling of the transmission range (one unit as aforementioned) and the the side length of the square region, we can have a connected wireless network with high probability. Hereafter, for simplicity, $UDG(V)$ is always assumed to be connected. We also assume that all wireless nodes have distinctive identities (IDs).

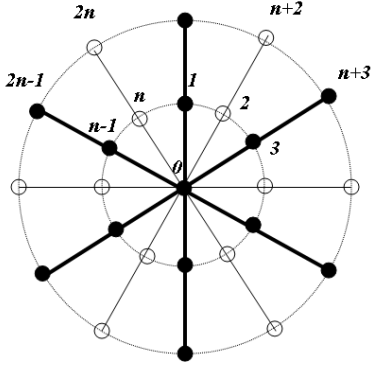
In the patent [1], an expanding ring algorithm is proposed to construct adjacency graph for reliable flooding of OSPF link state packets. The algorithm works as following: each node u tries to find another node with a smaller ID in its k -hop neighborhood, with k starting from 1 and increments of 1 in each round, until node u finds another node v such that $ID(v) < ID(u)$ or it has reached all nodes in the network. The latter case happens when node u has the smallest ID. If node u finds another node v such that $ID(v) < ID(u)$, then we denote this relationship as $u \rightarrow v$ and say u has an out-edge while v has an in-edge. It is not hard to see that the resulting graph, called Expanding Ring (ER) graph, is a shortest path tree rooted at the node with the smallest ID. This exemplary technique creates a spanning forest, with each tree in the forest comprising a shortest-path tree. Each of the shortest-path trees can then be linked together to form a tree rooted at the lowest numbered router. The OSPF broadcast performs by asking every non-leaf node to rebroadcast the message to its neighbors in the tree until all neighbors confirm it.

Lemma 1: The message complexity of broadcast based on Expanding Ring (ER) graph can be $O(n)$ times of the optimum, here n is the number of nodes in the network.

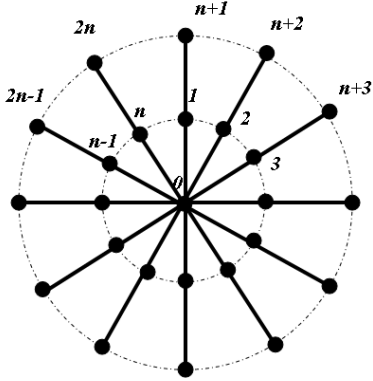
Proof: Figure 1 illustrates such a possible network topology: assume each node's transmission range is r , the inner circle's radius is r , and outer circle's radius is $2r$; there are $2n + 1$ nodes in the network, the center node ID is 0 (assume the node who has smallest id is 0), while the node IDs in inner and outer circle is $[1, n]$ and $[n + 1, 2n]$ respectively.

According to the expanding ring algorithm, every node in outer circle with ID $n + i$ has an out-edge to a node in the inner circle with ID i , and every nodes in the inner circle has an out-edge to the center node. In other words, the Expanding Ring graph consists of n rays centered at node 0, as illustrated in Figure 1 (b). Hence, the broadcast operation can be implemented by asking the node 0 and all nodes in inner circle to re-broadcast, while the outer circle nodes remain the same. Consequently, in the optimal case, the center node only needs to broadcast once, then each inner circle node needs to broadcast once for each, so the total broadcasting cost is $n + 1$.

On the other hand, the Minimum Connected Dominating Set (MCDS) on the same topology includes node 0 and $\pi / \arcsin(1/4)$ number of nodes at the outer circle as dominators, and $\pi / \arcsin(1/4)$ number of nodes at the inner circle as connectors, as illustrated in Figure 1 (a). The broadcast



(a) Minimum Connected Dominating Set (MCDS) graph



(b) Expanding Ring (ER) graph

Fig. 1. Broadcast cost of Expanding Ring graph can be as expensive as $O(n)$ times minimum broadcast cost.

operation can be implemented by asking every node in CDS to rebroadcast, while dominatees remains the same. Consequently, the total broadcast cost is $1 + 2\pi / \arcsin(1/4)$, which is a const number.

That is to say, in the worst case, broadcast message complexity of Expanding Ring graph is at least $\Theta(n)$ times of minimum. ■

This is disappointing news. However, slight modifications on the original algorithm can generate an expanding ring graph with backbone size bounded by a constant of $|MCDS|$. The modification are as following: (1) in the 1-hop expanding ring search, if a node already has an out-edge, it does not accept an in-edge. Notice that, after this step, all nodes without out-edges, during the 1-hop expanding ring search form a Maximal Independent Set (MIS). (2) the following k -hop expanding ($k > 1$) can use similar approach as original algorithm: every node without out-edge expands to search for another *dominator* node with smaller ID in its k hops. Finally, all nodes without in-edge form a Connected Dominating Set (CDS), called *ER-CDS* (Expended Ring based Connected Dominating Set) in the rest of the paper.

As follows, we will show that ER-CDS is a tree and its size is bounded by $12 \cdot OPT$, where OPT is the size (cardinality) of MCDS (Minimum Connected Dominating Set).

Lemma 2: ER-CDS is a tree topology.

Proof: To prove ER-CDS is a tree is to show that ER-

CDS is connected and loop-free.

First, we will show that ER-CDS is loop-free. Recall that, the modified 1-hop expanding ring search builds a Maximal Independent Set (MIS), because no other independent node can be added when the first step terminates. MIS is also a Dominating Set (DS). After that, each dominator u (e.g., the node without out-edges) expands to find one and only one dominator v , which has smaller ID than itself. And every dominatee only has one dominator, and no other dominatee will direct to this dominator, so there is no possibility that loop happen on such kind of nodes. For dominators, if loop happens, $ID(u_1) > ID(u_2) > \dots > ID(u_k) > ID(u_1)$. Here $>$ doesn't mean edge, it simply means to choose the right one as the dominator whose ID is smaller, so contradiction comes out, $ID(u_1)$ is smaller than $ID(u_1)$. Consequently, ER-CDS must be loop-free.

Then, we will show that ER-CDS is a connected. If it is not connected, then it must be a forest, since it is loop-free. For those tree roots, the only reason that they do not have a parent is because that there is no other node in the network with smaller ID. However, the node ID is unique and only the node with smallest ID cannot find a parent. There have been more than two nodes with smallest ID. Consequently, ER-CDS must be connected.

Hence, ER-CDS is a connected tree topology. ■

Let OPT be any Minimum Connected Dominating Set (MCDS) and let opt denote the size of OPT . For self-comprehensive presentation, we review the following lemma from [27]:

Lemma 3: [27] The size of any independent set in a unit-disk graph $G = (V, E)$ is at most $4opt + 1$.

Proof: Let U be any independent set of V , and let T' be any spanning tree of OPT . Consider an arbitrary pre-order traversal of T' given by v_1, v_2, \dots, v_{opt} . Let U_1 be the set of nodes in U that are adjacent to v_1 . For any $2 \leq i \leq opt$, let U_i be the set of nodes in U that are adjacent to v_i but none of v_1, v_2, \dots, v_{i-1} . Then U_1, U_2, \dots, U_{opt} form a partition of U . As v_1 can be adjacent to at most five independent nodes, $|U_1| \leq 5$. For any $2 \leq i \leq opt$, at least one node in v_1, v_2, \dots, v_{i-1} is adjacent to v_i . Thus U_i lies in a sector of at most 240 degree within the coverage range of node v_i (see Figure 3). This implies that $|U_i| \leq 4$. Therefore,

$$|U| = \sum_{i=1}^{opt} |U_i| \leq 5 + 4(opt - 1) = 4opt + 1$$

this completes the proof. ■

Theorem 4: The size of ER-CDS is bounded by $12 \cdot opt + 1$, where opt is the size of MCDS (Minimum Connected Dominating Set).

Proof: If there is a dominator of ER-CDS in OPT , then following similar proof of lemma 3, we can show that the total number of dominators is at most $1 + 4(opt - 1) = 4opt - 3$. Because ER-CDS is a tree and there are at most two connectors between adjacent dominators, the number of connectors is at most $2 \cdot 4(opt - 1) = 8opt - 8$. Hence $|ER-CDS| \leq 12opt - 11$.

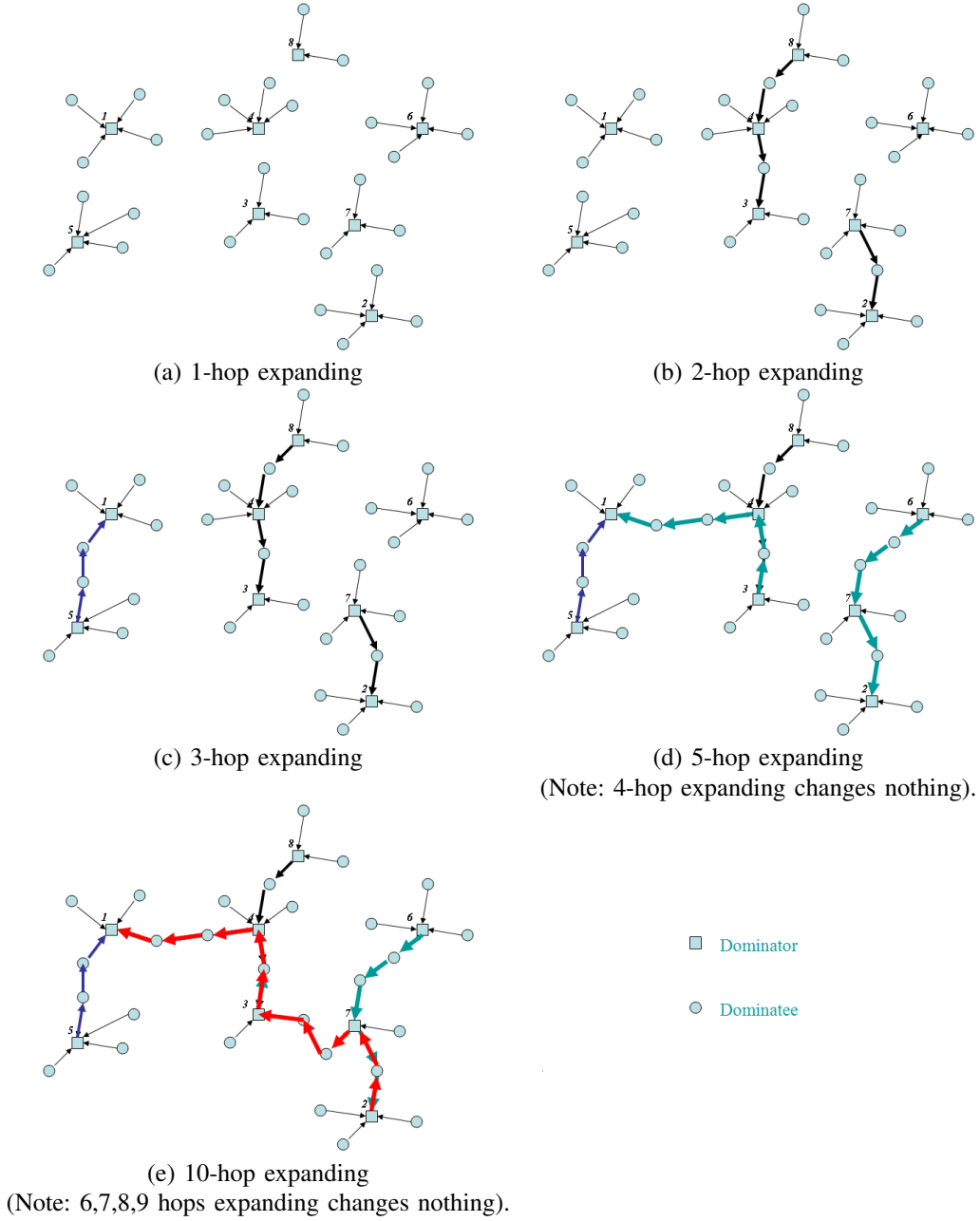


Fig. 2. Dogmatic expanding ring algorithm to construct ER-CDS.

Now, we assume there is no dominators in OPT . Let k be the number of dominators adjacent to the root of OPT . Similarly, as the proof of lemma 3, we can show that $k \leq 5$ and the total number of dominators in ER-CDS is at most $k + 4(opt - 1)$. Again, because ER-CDS is a tree and there are at most two connectors between adjacent dominators, the number of connectors is at most $2(4opt - 5 + k) = 8opt - 10 + 2k$. Hence $|ER - CDS| \leq 12opt - 14 + 3k \leq 12opt + 1$.

Overall, $|ER - CDS| \leq 12opt + 1$. ■

Notice that, smaller CDS means lower broadcast message complexity since nodes in CDS needs to rebroadcast messages.

III. SPLASH MERGING ALGORITHM FOR ER-CDS CONSTRUCTION

With the slight modification on the original expanding ring algorithm, the resulting ER-CDS graph has a constant bounded backbone size. However, the ER-CDS construction, based on modified expanding ring algorithm (as described in previous section), is still expensive. Some dominator nodes may need to broadcast multiple rounds of search messages before they finds another dominator with a smaller ID. An example of expanding ring procedure is illustrated in Figure 2. It takes 10-hop expanding-ring search before the ER-CDS forms, and 4,6,7,8,9 hops search did not change anything except wasting

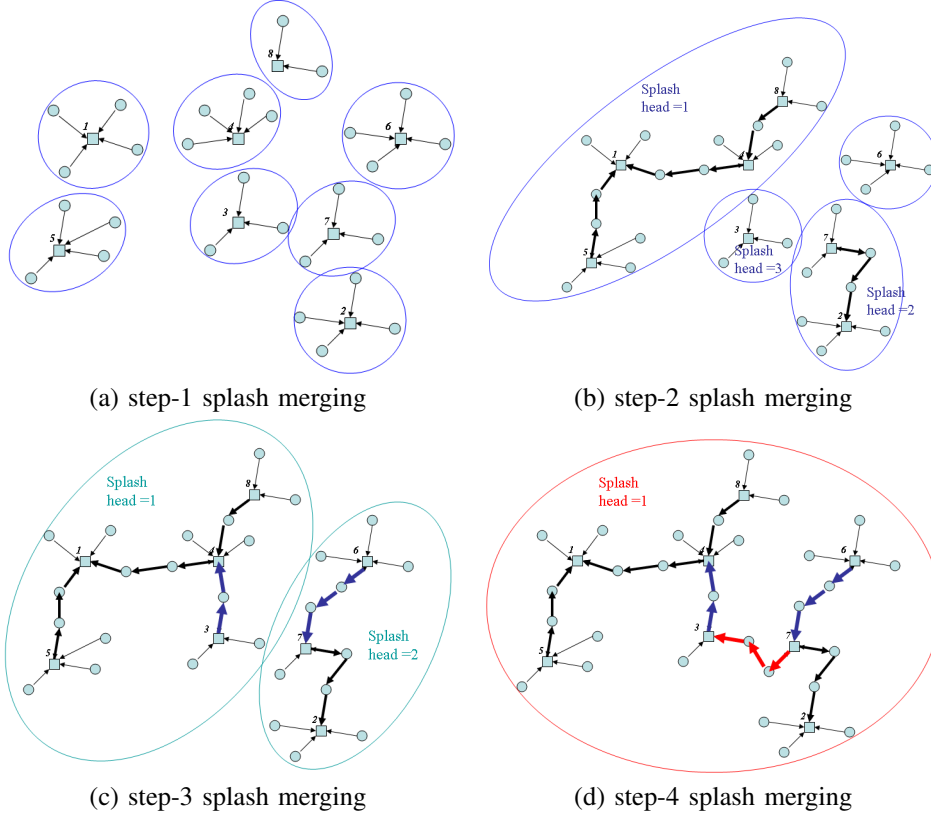


Fig. 4. Splash merging algorithm to construct ER-CDS.

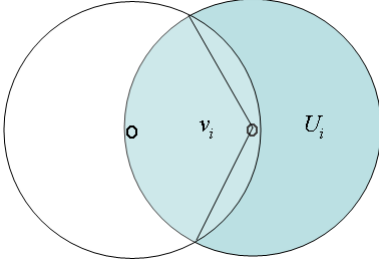


Fig. 3. U_i lie in a sector of at most 240 degree within the coverage range of node v_i .

energies.

In the section, we introduce an innovative splash merging algorithm to reduce the communication cost of constructing ER-CDS. In addition, it is not difficult to see that it also works for ER-CDS updates when topology changes, as it is a localized algorithm. The basic idea is to keep merging splashes (e.g., clusters) to bigger splashes, until it covers the whole network.

The following notations will be used in the rest:

- 1) $Size(u)$ is the number of dominatees of a dominator node u .
- 2) $Head(u)$ is the splash (e.g., cluster) head of node u .
- 3) $OldHead(u)$ is the previous splash (e.g., cluster) head of node u .
- 4) $AdjHead(u)$ is the adjacent splash head of node u . It

applies when node u is a potential connector.

- 5) $AdjNode(u)$ is the adjacent nodes of node u which belong to an adjacent splash. Both u and $AdjNode(u)$ could be potential connectors.
- 6) $I - AM - DOMINATOR$ is the message sent by the node which becomes dominator.
- 7) $I - AM - DOMINATEE$ is the message sent by the node which becomes dominatee.
- 8) $I - TRY - CONNECTOR$ is the message sent by the node that is the potential connector.
- 9) $UPDATE - HEAD$ is used to merge splash into new splash.

Algorithm 1 gives the splash merging algorithm basic ideas.

Figure 4 illustrates the procedure of splash merging algorithm on same network topology in Figure 2.

Theorem 5: The message and time complexity of ER-CDS formation algorithm 1 is $O(n + |DS||CDS|)$ and $O(|DS|)$ respectively. Here $|DS|$ and $|CDS|$ is the size of Dominator Set and Connected Dominating Set respectively.

Proof: In Step 1 of algorithm 1, each node sends two messages: one announces its ID, another announces its role as DOMINATOR or DOMINATEE. That is to say, the message and time complexity are $O(n)$ and $O(1)$ respectively.

In Step 2 of algorithm 1, the communication is along the CDS tree of each splash. In other words, only CDS nodes need to forward packets in each splash. And those CDS nodes participate in message forwarding for at most $|DS|$ times, as it

Algorithm 1 Splash Merging Algorithm for ER-CDS Construction

Step 1: Find dominators based on smallest ID.

- 1) Initially, every node u is marked as UNDECIDED, and set $Size(u) = 0$, $Head(u) = OldHead(u) = AdjHead(u) = AdjNode(u) = u$ and broadcast its own ID u .
- 2) If an UNDECIDED node u has smallest ID among its UNDECIDED neighbors, it will mark itself as DOMINATOR and broadcast a message I-AM-DOMINATOR(u , Head(u)).
- 3) If an UNDECIDED node v receives a message I-AM-DOMINATOR(u , w) and $Head(v) > u$, then node v will mark itself as DOMINATEE and record its dominator $Head(v) = u$, then broadcasts a message I-AM-DOMINATEE (v , Head(v)).
- 4) If a DOMINATOR node u receives a message I-AM-DOMINATEE (v , x) and $x == u$ for the first time, then it sets its $Size(u) = Size(u) + 1$.

Step 2: DOMINATOR node runs expanded ring algorithm to search connectors, until it finds and updates to C-DOMINATOR (e.g., Connected Dominator).

- 1) **Event:** a node v receives a message I-AM-DOMINATOR(u , w) or I-AM-DOMINATEE(u , w) or UPDATE(u, w, o, y, z) (if $o \neq headID(v)$) from u .
If $w < AdjHead(v)$ and $w < Head(v)$, then it sets $AdjHead(v) = w$, and $AdjNode(v) = u$. Basically, a node remembers potential connectors to the closest splash (e.g., cluster) with smallest head ID. If a node v finds its $AdjHead(v) < Head(v)$, then sends a message I-TRY-CONNECTOR (v , Head(v), $AdjHead(v)$, v , $AdjNode(v)$) towards the splash head.
- 2) **Event:** a cluster head v receives a message I-TRY-CONNECTOR (x , u , w , y , z).
If $Head(v) == u$ and $w < AdjHead(u)$, then it sets $AdjHead(v) = w$;
if $v \neq Head(v)$, send a message I-TRY-CONNECTOR (v , Head(v), $AdjHead(v)$, y , z);
if $v == Head(v)$, set $OldHead(w) = Head(w)$, $Head(v) = w$, updates to C-DOMINATOR and broadcasts a message UPDATE-HEAD(v , Head(v), $OldHead(v)$, y , z).
- 3) **Event:** node w receives a message UPDATE-HEAD(x, u, o, y, z).
If $Head(w) == o$, then it sets $OldHead(w) = Head(w)$, $Head(w) = u$;
if $Head(w) == o$, $w == y$ and w is DOMINATEE, then it sets itself as CONNECTOR;
if $Head(w) == o$, $w == z$, w is DOMINATEE, then it sets itself as CONNECTOR. Broadcasts a message UPDATE-HEAD(w , Head(w), $OldHead(w)$, y , z).

Finally, the C-DOMINATOR and CONNECTOR forms the backbone of a network, called ER-CDS in the following presentation.

takes at most $|DS|$ steps to connect all dominators. Hence, the message complexity is $O(|CDS||DS|)$, while time complexity is $O(|DS|)$.

Consequently, the message complexity of algorithm 1 is $O(n + |DS||CDS|)$, where $|DS|$ is the number of dominators. Usually, $|DS|$ and $|CDS|$ is much smaller than n , it means that the message complexity is actually $O(n)$. And, the time complexity is $O(|DS|)$. ■

In [27], [29]. Wan *et al.*, proposed an algorithm to construct a connected dominating set distributively as a backbone for a wireless network in [27]. Their algorithm has an approximation factor of at most 8, $O(n)$ time complexity and $O(n \log n)$ message complexity for a wireless network with size n . Their main idea is to construct the tree first, based on which CDS is constructed. Notice that, to start the tree construction, it needs a separate leader election phase before it, which is at least $n \log n$ ($O(n^2)$ in worst case). This is known to have the lowest approximation ratios among distributed methods. By the way, though later the work [2] has lower approximation ratio, it is a centralized greedy algorithm, which is out of our interest for comparison.

Our ER-CDS graph has a constant bound of MCDS, though its approximation factor is not as low as [27]. More importantly, the ER-CDS is constructed in fully localized algorithm with lower time and message complexity, which is more important for a dynamic environment, as lower time and message complexity means faster and lower-cost update when topology changes. In [27], the CDS construction depends on leader election and tree maintenance, some topology changes may cause the CDS to reconstruct. In other words, the topology updates are not necessarily localized. In the next section, we will also show that the reliable flooding based ER-CDS is also lower than the CDS in [27].

IV. PERFORMANCE EVALUATION

We conduct extensive simulations to evaluate the performance of ER-CDS via TOSSIM on TinyOS 2.0, by comparing it to the algorithm in [27].

A. Simulation Environment

We use TinyOS-2.x TOSSIM to conduct the simulations. To compare the various performances between ER-CDS and Wan's algorithm in [27], we vary network size from 30 to 210 with step 30, while keeping the network density unchanged. For each pair of nodes that fall in the transmission range of each other, we randomly generate the link quality between them in order to simulate the realistic situation. The evaluation compared the following four important performance metrics:

- **Dominator Size:** the number of dominators
- **Connector Size:** the number of connectors
- **Setup Cost:** the number of messages used to construct the CDS.
- **Broadcast Cost:** the number of messages for reliable flooding of link state packets.

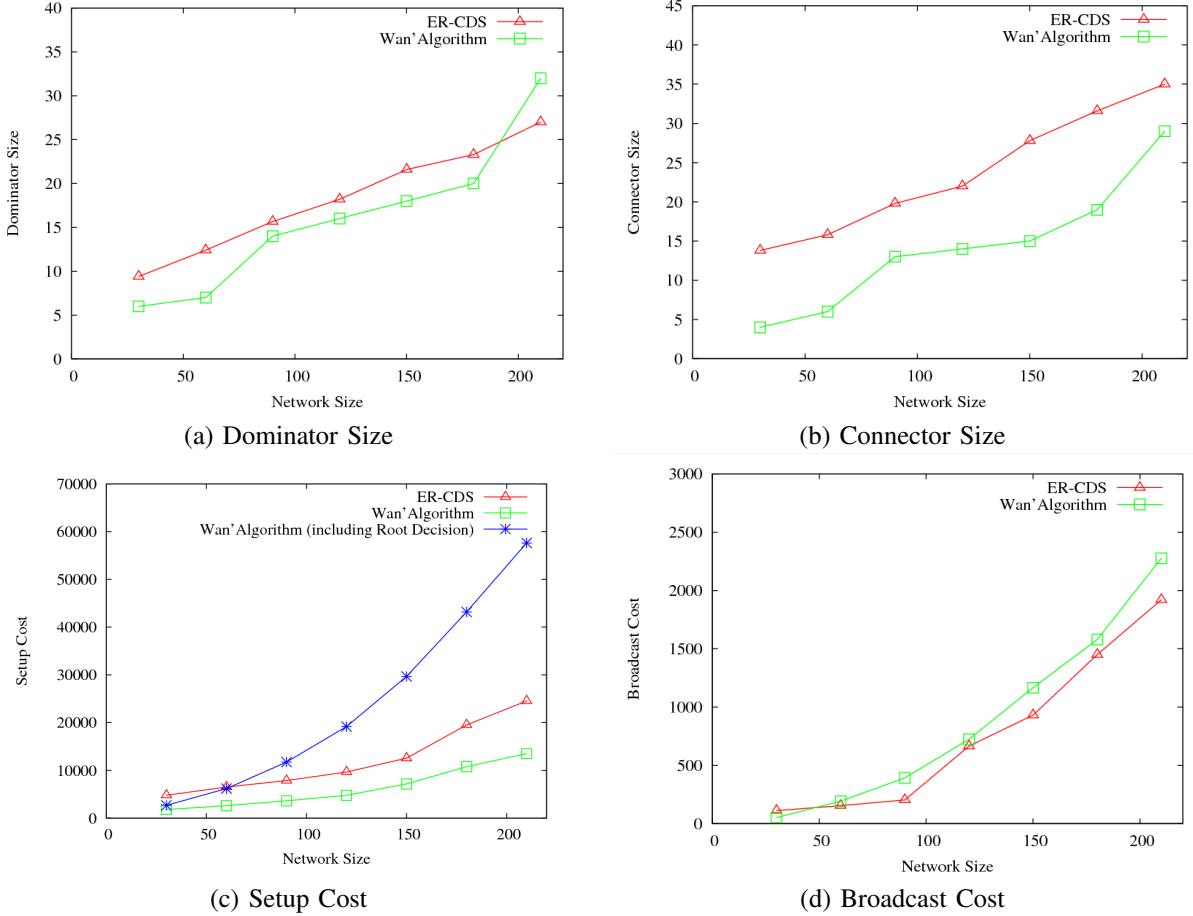


Fig. 5. Compare Simulation result between ER-CDS and Wan's approach.

As mentioned at the beginning of this paper, OSPF has conventionally been implemented in wired point-to-point or multi-access networks. And to implement OSPF over a multi-hop, multi-access packet radio network with its own private, internal routing system, so as to permit seamless integration of such a network into an OSPF environment. However many problems arise when it is moved to a wireless network. Our target is to build a CDS, and realize reliable flooding in this network through backbone composed of dominators and connectors. Then we collect and count the cost as follows.

Dominators and connectors rebroadcast packets to ensure that packets can reach every node in the network, while dominatees only receive packets but not forward them. After the CDS is built, the dominators will remember the information of their neighbors, which includes some useful properties, such as how many dominatees and connectors connected to them. Hence, they can make broadcast reliable by checking whether ACKs are received from intended neighbors; if not, a dominator shall rebroadcast again periodically.

B. Simulation Results

We evaluate two protocols under different random topologies. Figure 5(a) shows that when the network size is not too big, the dominator size of Wan's Algorithm gets smaller

than that of ER-CDS. With the increment of the network size, dominator size increases for both of algorithms, but ER-CDS increases slower than CDS. So at this point, ER-CDS performs better. The reasons for this difference are observed as follows.

The way to choose Dominators has been described in the step 1 described in the algorithm 1. For ER-CDS, a dominator is decided by its ID, while a dominator is decided by nodes' levels (which are determined in the neighbor discovering step) in Wan's algorithm [27]. After the neighbor discovery, the root node begins to use coloring method to find dominator. For ER-CDS, the uncolored node with smallest ID is more possible to be selected as a dominator, and in Wan's algorithm, the node with the lowest rank ($\langle level, ID \rangle$) has a greater chance to become a dominator. As we can see from the Fig. 5(a), ER-CDS has more advantages When it comes to bigger network size.

Figure 5(b) shows that the trend of connector size with the increment of the network size. As illustrated in the Fig. 5(b), the ER-CDS's connector size is bigger than the other one. This meets our conjecture and we analyze the reasons. The connector size for a graph is directly related with the dominator size. After the dominators are chosen, the dominator will choose some dominatees to form a connected dominating set. According to Wan's algorithm, they have a smaller connector

size since the construction of CDS is based on a tree structure and they guarantee that there is at most one connector for adjacent dominator pair.

Figure 5(c) shows that setup cost (total messages used to construct CDS) of both algorithms. The setup cost used by ER-CDS is bigger than that of Wan's algorithm. This is because ER-CDS chooses dominators and connectors by two steps while the dominators and connectors are chosen in the same step in Wan's algorithm. In the first step, for both algorithm, all the nodes discover their neighbors first by collecting information from one-hop neighbors. According to Wan's algorithm, it will find dominators and connectors at the same time since every non-leaf dominatee node will be selected as a connector automatically based on the tree structure; however, for ER-CDS, it only can find dominators by smallest IDs during the first step. In the second step of ER-CDS, every node will update its HeadID round by round until all nodes form a connected single cluster. This part of cost leads to bigger setup cost of ER-CDS. One thing that needs to be mentioned is that in the implement of Wan's Algorithm, it assumes the node which has smallest ID will be selected as a root node. This is not true in most of cases, and to choose the root also consumes setup resources. This is because every node has to broadcast information (messages) in order to know the information of the whole network. It has been proved in [31] that for a wireless network with size n , to find the root node (or say electing the leader) has message complexity $O(n^2)$ in worst case and $O(n \log n)$ message complexity in average case. The total construction cost of Wan's algorithm including leader election is also illustrated in Fig. 5(c).

The trend of broadcast cost for both algorithms has been shown in Fig. 5(d). From the result, we can see that with the increment of the network size, ER-CDS has less broadcast cost. The measurement of broadcast cost is conducted as follows. First the root starts to broadcast a packet, then for each dominator or connector, it will rebroadcast the packet after it received the packet. When a dominatee receives a packet, it will send ACK back to the dominator who sends the packet. Next, only if a node finds that it has received all the ACKs from the nodes that connect to root through it, it will reply ACK back to the upper layer repeatedly until all nodes have a copy of the packet.

Clearly, the broadcast cost depends on dominator size and original topology. Because ER-CDS selects dominators based on IDs, the topology and ID distribution have a great effect on the broadcast cost. One of methods to reduce the cost is to adjust the node ID, such as decreasing the dominator size. For example, if we have a simple network with size 3 that forms a line graph, in which node 2 is the intermedia node between node 1 and 3. Based on ER-CDS discipline, we will choose node 1 and 3 as dominators. However, if we could change node's ID, for example we switch IDs of node 1 and 2, then the only dominator will be node 1. This is an advantage of ER-CDS since Wan's Algorithm [27] didn't only depend on nodes' ID. In addition, in Wan's algorithm, the cost of finding proper root and the dominators is fixed, but in ER-CDS, there

may be some possibility to change partial topology such that the whole network dominator size is decreased.

One thing that needs to be mentioned is that we did not run simulation to compare our algorithm with the patent method in [1] since its communications cost is clearly higher than our ER-CDS algorithm. In addition, the patent method needs to include the full path information to locate connectors in the message, which may exceed the message size limit of TinyOS (e.g., at most 128 bytes at 802.15.4 radio).

V. RELATED WORKS

After first noted by Ephremedis *et al.*, in [14] that a CDS can create a virtual network backbone for packet routing and control. Many researchers have proposed a bunch of CDS related construction methods and application related algorithms in order to improve the performance for wireless networks by using CDS. Basically, algorithms that construct a CDS in ad hoc networks can be divided into two categories: centralized algorithms that depend on network-wide information or coordination and decentralized that depend on local information only. Guha and Khuller proposed two CDS construction algorithms in their seminal work [15] in 1998, in which they proposed two greedy heuristic algorithms with bounded performance guarantees. In the first algorithm, the CDS is grown from one node outward. In the second algorithm, they first constructed a weighted CDS, and then intermediate nodes were selected to create a CDS. In addition, Ruan *et al.*, in [16] gave a one-step greedy approximation algorithm by coloring with performance ratio at most $3 + \ln(\Delta)$. Here, Δ is the maximum degree of the the communication graph. An interesting work has been done by Cheng *et al.*, [17], in which they proposed a greedy algorithm for minimal CDS in unit-disk graphs. Their algorithm relies on an maximal independent set (MIS) but the resultant CDS may not contain all the elements in the MIS. Recently Min *et al.*, in [18] propose to use a Steiner tree with minimum number of Steiner nodes (ST-MSN) to connect a maximal independent set before computing CDS. However, all of above mentioned algorithms are centralized based which is not suitable for large scale wireless networks, in which decreasing the communication cost is one of important objectives.

In pure localized algorithms (or distributed algorithms), which is more realistic for wireless networks, the status of each node depends on its k -hop topology only, where k is a small constant, and usually converges after at most k rounds of information exchange among neighbors. Chen *et al.*, propose a series of approximate algorithms for computing a small WCDS to be used to cluster mobile ad hoc networks in [19]. Das *et al.*, in [20] implemented both algorithms in the distributed pattern proposed by Guha and Khuller in [15]. Alzoubi and Wan's in [27], [28] utilize the properties of unit-disk graphs (UDGs) in which they provided two versions of an algorithm to construct the dominating set for a wireless network. In both algorithms, they first constructed a rooted spanning tree from the original network topology using the distributed leader election algorithm. Then, based on the levels

of nodes in the tree, they divided nodes into different ranks and further get a CDS with the time and messages complexity $O(n)$ and $O(n \log n)$ respectively. In their scheme, a maximal independent set (MIS) is elected such that each vertex in the MIS can be connected to the spanning tree via an extra vertex. Since, in unit disk graphs, the size of an independent set is at most 4 times that of the minimum CDS, this algorithm has an approximation ratio of 8. However, this algorithm usually produces a larger CDS than the MCDS algorithm in random unit disk graphs. Later, Cheng introduced two algorithms for growing a connected dominating set from a leader node in [21], [22]. Compared with the work of Alzoubi and Wan's works in [27], [28], they introduce a new active state for vertices to describe the current labeling set of vertex nodes.

Another type of well known construction methods of CDS is based on pruning. Wu *et al.*, proposes a completely localized algorithm to construct CDS in general graphs in [23]. In other works, such as in [23]–[26], each node determines its own status and is in the forward status by default. A node resigns its role (dominators, connectors and dominatees) of forward status by itself if a path from the source can be found for each of its neighbors. Nodes in such a path can be either already forwarded nodes or nodes that deem to forward.

VI. CONCLUSION

In this paper, topology control algorithms to support efficient OSPF link state routing have been studied. An existing patent method [1] proposes an expanding ring algorithm to build adjacency graph for reliable flooding. We first analyze this method and show the message complexity of reliable flooding based on it could be $O(n)$ times of optimum. We then show that a slight modification on it can generate a adjacency graph, called ER-CDS, which supports reliable flooding with message complexity $O(1)$ of optimum. However, the construction cost based on the patent method is still high and does not adapt to network topology changes well. Finally, we propose a localized splash merging algorithm to construct ER-CDS, and conducted extensive simulations to evaluate its performance.

REFERENCES

- [1] JOSEPH J. WEINSTEIN, VLADIMIR ROSENZWELG AND JOSEPH KELLER Systems and Methods For Forming An Adjacency Graph for Exchanging Network Routing Data In *U.S. Patent Application Publication US 2005/0041676 A1*, Feb. 24, 2005.
- [2] YINGSHU LI, MY T. THAI, FENG WANG, CHIH-WEI YI, PENG-JUN WAN AND DING-ZHU DU On Greedy Construction of Connected Dominating Sets in Wireless Networks In *Wireless Communications Mobile Computing*, Volume 5, Issue 8, December 2005
- [3] JEREMY BLUM, MIN DING, ANDREW THAELER AND XIUZHEN CHENG Connected Dominating Set in Sensor Networks and MANETs In *Handbook of Combinatorial Optimization* (329369, 2006).
- [4] B. AN AND S. PAPAVALSILIOU A mobility-based clustering approach to support mobility management and multicast routing in mobile ad-hoc wireless networks In *International Journal of Network Management* (387395, 2001).
- [5] M. GERLA AND J. T. TSAI M. Gerla and J. T. Tsai In *ACM/Baltzer Journal on Wireless Networks* (255-265, 1995).
- [6] S. DATTA AND I. STOJMENOVIC Internal Node and Shortcut Based Routing with Guaranteed Delivery in Wireless Networks In *Cluster Computing* (5(2):169-178, 2002).
- [7] B. CHEN, K. JAMIESON, H. BALAKRISHNAN, AND R. MORRIS An energy-efficient coordination algorithm for topology maintenance in Ad Hoc wireless networks In *Wireless Networks* (481-494, 2002).
- [8] J. SHAIKH, J. SOLANO, I. STOJMENOVIC, AND J. WU New Metrics for Dominating Set Based Energy Efficient Activity Scheduling in Ad Hoc Networks In *Proc. of WLN Workshop* (726-735, Oct. 2003).
- [9] Y. XU, J. HEIDEMANN, AND D. ESTRIN Geography-informed energy conservation for Ad Hoc routing In *ACM MobiCom* (70-84, 2001).
- [10] BUDHADITYA DEB, SUDEPT BHATNAGAR, AND BADRI NATH Multi-resolution State Retrieval in Sensor Networks In *IEEE Workshop on Sensor Network Protocols And Applications (SNPA)* (2003).
- [11] M. DING, X. CHENG, AND G. XUE Tag: a tiny aggregation service for ad-hoc sensor networks In *Proc. of IEEE VTC* (2003).
- [12] D.B. JOHNSON AND D.A. MALTZ Dynamic source routing in ad hoc wireless networks In *Proc. Mobile Computing* (1996).
- [13] C. PERKINS AND E. ROYER Ad-hoc on-demand distance vector routing In *Proc. 2nd IEEE Workshop on Mobile Computing Systems and Applications* (90-100,1999).
- [14] A. EPHREMIDES, J. WIESELTHIER, AND D. BAKER A design concept for reliable mobile radio networks with frequency hopping signaling In *Proceedings of the IEEE* (56-73, 1987).
- [15] S. GUHA AND S. KHULLER Approximation algorithms for connected dominating sets In *Algorithmica* (36(SI):374-387, 1998).
- [16] L. RUAN, D.H. DU, X. JIA, W. WU, Y. LI, AND K.-I KO A greedy approximation for minimum connected dominating sets In *Theor. Comput. Sci.* (329, 1-3 ,325-330,Dec. 2004).
- [17] X. CHENG, M. DING, AND D. CHEN An approximation algorithm for connected dominating set in ad hoc networks In *Proc. of International Workshop on Theoretical Aspects of Wireless Ad Hoc, Sensor, and Peer-to-Peer Networks (TAWN)* (2004).
- [18] MANKI MIN AND HONGWEI DU AND XIAOHUA JIA AND CHRISTINA XIAO HUANG AND SCOTT C.-H. HUANG AND WEILI WU Improving Construction for Connected Dominating Set with Steiner Tree in Wireless Sensor Networks In *J. of Global Optimization* (2006).
- [19] Y. CHEN, A. LIESTMAN Approximating minimum size weakly-connected dominating sets for clustering mobile ad hoc networks In *ACM MobiHoc* (2002).
- [20] BEVAN DAS AND VADUVUR BHARGHAVAN Routing in Ad-Hoc Networks Using Minimum Connected Dominating Sets In *International Conference on Communications* (1997).
- [21] S. BUTENKO, X. CHENG, D.-Z. DU, AND P.M. PARDALOS On the construction of virtual backbone for ad hoc wireless networks In *Cooperative Control: Models, Applications and Algorithms* (4354, 2003).
- [22] X. CHENG Routing Issues in Ad Hoc Wireless Networks In *PhD Thesis* (2002).
- [23] J. WU AND H. LI On Calculating Connected Dominating Set for Efficient Routing in Ad Hoc Wireless Networks In *Proc. of the Third International Workshop on Discrete Algorithms and Methods for Mobile Computing and Communications* (714, 1999).
- [24] B. CHEN, K. JAMIESON, H. BALAKRISHNAN, AND R. MORRIS Span: an energy-efficient coordination algorithm for topology maintenance in ad hoc wireless networks In *ACM Wireless Networks Journal* (481-494,2002).
- [25] W. PENG AND X.C. LU On the reduction of broadcast redundancy in mobile ad hoc networks In *MobiHoc* (129130, 2000).
- [26] I. STOJMENOVIC, S. SEDDIGH, AND J. ZUNIC Dominating sets and neighbor elimination based broadcasting algorithms in wireless networks In *Trans. on Parallel and Distributed Systems* (14-25, 2002).
- [27] P.-J. WAN, K.M. ALZOUBI AND O. FRIEDER Distributed Construction of Connected Dominating Set in Wireless Ad Hoc Networks In *Mobile Networks and Application* (2004).
- [28] K.M. ALZOUBI, P.-J. WAN AND O. FRIEDER New Distributed Algorithm for Connected Dominating Set in Wireless Ad Hoc Networks In *Proceedings of the 35th Hawaii International Conference on System Sciences* (2002).
- [29] XIANG-YANG LI, GRUIA CALINESCU, PENG-JUN WAN, YU WANG Localized Delaunay Triangulation with Applications in Wireless Ad Hoc Networks, In *IEEE Transactions on Parallel and Distributed Systems (TPDS)* (Vol. 14, No. 10, 1035- 1047, Oct. 2003).
- [30] XUFEI MAO, XIANG-YANG LI AND S. KAMI MAKKI Static Channel Assignment for Multi-channel Multi-hop and Multi-radio Wireless Networks In *International Workshop on Theoretical and Algorithmic Aspects of Sensor and Ad-Hoc Networks (WTASA07)* (2007).

- [31] I. CIDON AND O. MOKRYN Propagation and leader election in multihop broadcast environment In *Proc. of 12th International Symposium on Distributed Computing (DISC98), Greece (September 1998)* pp. 104C-119./ (1998).