Efficient Interference-Aware TDMA Link Scheduling for Static Wireless Networks

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ABSTRACT

We study efficient *link scheduling* for a multihop wireless network to maximize its throughput. Efficient link scheduling can greatly reduce the interference effect of close-by transmissions. Unlike the previous studies that often assume a unit disk graph model, we assume that different terminals could have different transmission ranges and different interference ranges. In our model, it is also possible that a *communication link* may *not* exist due to barriers or is not used by a predetermined routing protocol, while the transmission of a node always result interference to *all* non-intended receivers within its interference range. Using a mathematical formulation, we develop synchronized TDMA link schedulings that optimize the networking throughput. Specifically, by assuming known link capacities and link traffic loads, we study link scheduling under the RTS/CTS interference model and the protocol interference model with fixed transmission power. For both models, we present both efficient centralized and distributed algorithms that use time slots within a constant factor of the optimum. We also present efficient distributed algorithms whose performances are still comparable with optimum, but with much less communications. Our theoretical results are corroborated by extensive simulation studies.

Categories and Subject Descriptors

C.2.1 [**Network Architecture and Design**]: Wireless communication; G.2.2 [**Graph Theory**]: Network problems, Graph algorithms.

General Terms

Algorithms, Design, Theory.

Keywords

Link scheduling, Interference, Graph Coloring, Distributed Algorithm, Wireless Networks.

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1. INTRODUCTION

Wireless multi-hop radio networks such as ad hoc, mesh, or sensor networks are formed of autonomous nodes communicating via radio. Wireless networks draw lots of attentions in recent years due to their potential applications in various areas. For example, wireless mesh networks are being used as the last mile for extending the Internet connectivity for mobile nodes. These networks behave almost like wired networks since they have infrequent topology changes, limited node failures, *etc.*. For wireless mesh networks or sensor networks, the aggregate traffic load of each routing node changes infrequently also. A unique characteristic of wireless networks is that the radio sent out by a wireless terminal will be received by all the terminals within its transmission range, and also possibly causes signal interference to some terminals that are not intended receivers. In other words, the communication channels are shared by the wireless terminals. Thus, one of the major problems facing wireless networks is the reduction of capacity due to interference caused by simultaneous transmissions. Using multiple channels and multiple radios can alleviate but not eliminate the interference. To achieve robust and collision free communication, there are two alternatives. One is to utilize a random access MAC layer scheme. The other is to carefully construct a transmission schedule. One variant, link scheduling in the context of time division multiplexing (TDM) is the subject of this paper.

In this paper, we assume that the time is slotted and synchronized. A link scheduling is to assign each link a set of time slots $\subset [1, T]$ on which it will transmit, where T is the scheduling period. A link scheduling is *interference-aware* (or called *valid*) if a scheduled transmission on a link $x \rightarrow y$ will not result in a collision at either node x or node y (or any other node). In this context, two types of collisions must be avoided, namely, primary interference and secondary interference. Link scheduling has received a great attention from both networking and theory fields [1, 16–21, 23, 26] in the past few years due to its application for assigning time slots in TDMA MAC protocols that eliminate collision, guarantee fairness. Many scheduling problems in wireless networks have been shown to be NP-complete, including TDMA broadcast scheduling [7], link scheduling [2, 8]. For some of these problems, even polynomial-time algorithms with constant approximation ratios appear unlikely for general graphs.

Previous studies on link scheduling either assume a very general graph model or assume a very specific graph model such as unit disk graph (UDG). It is widely accepted in the wireless networking community that neither a general graph model nor UDG model accurately captures unique properties of wireless networks. A general graph model could not capture a certain geometry property of wireless networks, *e.g.*, two nodes must be within certain distance to be able to communicate directly (or one node's transmission could interfere the other node's reception). A unit disk graph model is idealistic since in practice two nearby nodes may still be unable to communicate due to various reasons such as barrier and path fading. In this paper, we give efficient centralized and distributed algorithms to obtain a valid link scheduling with theoretically proven performances for a more realistic wireless network model. The main contributions of this paper are as follows.

- **More Realistic Model:** We address the link scheduling in a more realistic networking model: (1) each node has its own transmission power and thus its own transmission range; (2) that the receiver must be within the transmission range of the sender is only a necessary (but not sufficient) condition for two nodes to communicate directly, *i.e.*, two nearby nodes may still be unable to communicate directly; (3) if a node v is within certain distance of a sender u, then the transmission by u will interfere the reception of node v. In summary, the communication graph could be an arbitrary geometry graph. Notice that similar realistic models using weighted and unweighted flows, modeling interference range to be different from transmission range, etc. have all been proposed and modeled in earlier work, *e.g.* in [15, 18, 21], and heuristic algorithms have been given for each or all of these. Our contributions here are that we provide theoretical bounds for link-scheduling algorithms in these cases.
- **Both Weighted and Unweighted Flow:** In several wireless networks (*e.g.*, mesh, sensor networks), we can estimate the traffic demand by each wireless node. Thus, based on a given routing algorithm, we can predict the average traffic load $\ell(e)$ on each link e of the network. We then design link scheduling algorithms to meet this traffic demand if possible. We model this by assuming that each link e has an integral *weight* $w(e)$ specifying the number of slots it needed in a period to support its traffic load. Here $w(e) = \lceil T \cdot \frac{\ell(e)}{c(e)} \rceil$, where $c(e)$ is the capacity of link e if there is no interference, and T is a given period for a schedule. In certain networks, it is difficult, if not impossible, to estimate the load of every link. We then assume that each node needs at least one time slot for transmission and our objective is to design a scheduling that minimizes T.
- **Theoretical Performance Guarantee for Efficient Centralized/Distributed Algorithms:** We consider two kinds of interference models: RTS/CTS model and protocol interference model with fixed transmission power. For both models, we present both centralized and distributed link scheduling algorithms that use time slots at most a constant factor of the optimum. All algorithms involve a novel study of interference properties in wireless networks. For the protocol interference model, we require that the interference range of a node is larger than its transmission range, which is always true in practice (the interference range of a node is about twice of its transmission range). One of our distributed algorithms has not only small communication complexity, but also good performance guarantee that is only logarithmic of the ratio between the maximum and minimum interference range. Although some of our algorithms are similar to some algorithms proposed before, to the best of our knowledge, we are the first one to prove asymptotical optimal bounds for the performance. We also present both necessary and sufficient conditions for schedulable flows under interference.
- **Layer Independence:** To preserve the independence between layers, we assume that there is already an existing routing algorithm that will select a path for every pair of source and destination nodes. The performance guarantee of methods presented here is *independent* of the routing algorithm when the routing is given. The results presented here can also be extended to the scenario when we want to maximize the throughput by optimizing the routing and TDMA link scheduling together.

The rest of the paper is organized as follows. In Section 2, we discuss the network models and interference models and formally define the problem studied in this paper. We present our centralized algorithms for link scheduling in Section 3. We also analyze the theoretical guaranteed performances of our algorithms. Our distributed algorithms are presented in Section 4. In Section 5, we study how to assign time slots to links when each link has a requirement of the least number of time slots needed. Our simulation studies are reported in Section 6. In Section 7, we briefly review the related works in the literature. We conclude our paper in Section 8.

2. SYSTEM MODEL AND ASSUMPTIONS

Interference issues have been studied extensively recently because it is widely believed that reducing the interference can increase the overall performance of a wireless network. There are different approaches to reduce the interference, including the scheduling on the MAC layer, route selection on the routing layer and power control on the physical layer. In this section, we first discuss in detail the interference models we will use and formally define the problem that we will study in this paper.

2.1 Network and Interference Models

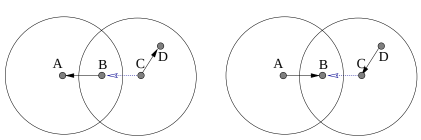
NETWORK MODEL: In this paper, we assume that there is a set V of communication terminals deployed in a plane. Each wireless terminal is only equipped with *single* radio interface. The complete communication graph is a *directed* graph $G = (V, E)$, where $V = \{v_1, \dots, v_n\}$ is the set of terminals and E is the set of possible directed communication links. Every terminal v_i has a transmission range t_i such that the necessary condition for a terminal v_j to receive correctly the signal from v_i is $\|v_i - v_j\| \le t_i$, where $\|v_i - v_j\|$ (sometimes we denote it as $d_{i,j}$ for simplicity) is the Euclidean distance between v_i and v_j. Notice that $\|v_i - v_j\| \le t_i$ is not the sufficient condition for $(v_i, v_j) \in E$. Some links do not belong to G because of either the physical barriers or the selection of routing protocols. This is the major distinction of our model with the majority previous studies on link scheduling. To the best of our knowledge, only [21] used the similar model as ours. We always use $l_{i,j}$ to denote (v_i, v_j) hereafter. Each terminal v_i also has an interference range r_i such that v_j is interfered by the signal from v_i if $\|v_i - v_j\| \le r_i$ and v_j is not the intended receiver. The interference range r_i is not necessarily same as the transmission range t_i. Typically, $r_i > t_i$. We call the ratio between them as the *Interference-Transmission Ratio* for node v_i, denoted as $\gamma_i = \frac{r_i}{t_i}$. In practice, $2 \le \gamma_i \le 4$. For all wireless nodes, let $\gamma = \max_{v_i \in V} \frac{r_i}{t_i}$.

INTERFERENCE MODELS: To schedule two links at the same time slot, we must ensure that the schedule will avoid the interference. Two different types of interference have been studied in the literature, namely, *primary interference* and *secondary interference*. Primary interference occurs when a node transmits and receives packets at the same time. Secondary interference occurs when a node receives two or more separate transmissions. Here all transmissions could be intended for this node, or only one transmission is intended for this node (thus, all other transmissions are interference to this node). In addition to these interferences, there could have some other constraints on the scheduling, *e.g.*, the radio networks that deploy the IEEE 802.11 protocol with request-to-send and clear-to-send (RTS/CTS) mechanism will pose some additional constraints. Several different interference models have been used to model the interferences in wireless networks. We briefly review them in the following.

Protocol Interferences Model (PrIM): It was first proposed in [13]. In this model, a transmission by a node v_i is successfully received by a node v_j iff the intended destination v_j is sufficiently apart from the source of any other simultaneous transmission, *i.e.*, $\|v_k - v_j\| \geq (1+\eta)\|v_i - v_j\|$ for any node $v_k \neq v_i$. Here constant $\eta > 0$ models situations where a guard zone is specified by the protocol to prevent a neighboring node from transmitting on the same channel at the same time. This model *implicitly* assumed that each node v_k will adopt the power control mechanism when it transmits signals. Simulation analysis [12] as well as the analytical results [3] indicate that the protocol interference model does not necessarily provide a comprehensive view of reality due to the aggregate effect of interference in wireless networks. However, it does provide some good estimations of interference and most importantly it enables a theoretical performance analysis of a number of protocols designed in the literature. Link scheduling using PrIM interference model and network model similar to ours has been studied in [21].

Fixed Power Protocol Interferences Model (fPrIM): We adopt the following interference model throughout this paper. We assume that a node will *not* dynamically change its power based on the intended receiver in a packet-level. Note that this assumption does not preclude the power control that can further reduce the power consumption. We only assume that there is no power adaptation at the packet level and the power is not adjustable for a certain period of time, which is close to the real situation. However, we do assume that each node v_i has its own fixed transmission power and thus a fixed transmission range t_i. We also assume that each node v_k has an *interference range* r_k such that any node v_j will be interfered by the signal from v_k if $\|v_k - v_j\| \leq r_k$ and node v_k is sending signal to some node other than v_j. In other words, the transmission from v_i to v_j is viewed successful if $\|v_k - v_j\| > r_k$ for every node v_k transmitting in the same time slot using the same channel.

RTS/CTS Model: This model was also studied previously, *e.g.*, [1]. For every pair of transmitter and receiver, all nodes that are within the interference range of either the transmitter or the receiver cannot transmit. Figure 1(a) shows the case that communication from B to A and C to D cannot take place simultaneously due to RTS. Figure 1(b) shows the case that communication from A to B and D to C cannot take place simultaneously due to CTS. Although RTS/CTS is not the interference itself, for convenience of our notation, we will treat the communication restriction due to RTS/CTS as *RTS/CTS interference* model. Thus, for every pair of simultaneous communication links, say $v_i v_j$ and $v_p v_q$, it should satisfy that (1) they are distinct four nodes, *i.e.*, $v_i \neq v_j \neq v_p \neq v_q$; (2) v_i and v_j are not in the interference ranges of v_p and v_q, and vice versa. The *interference region*, denoted by $I_{i,j}$, of a link $l_{i,j}$ is the union of the interference region of nodes v_i and v_j. When a directed link $v_i v_j$ (or $v_j v_i$) is active, all simultaneous transmitting links $v_p v_q$ cannot have an end-point inside the area $I_{i,j}$. Notice, it is possible that neither v_p nor v_q is in $I_{i,j}$ but $l_{p,q}$ still interferes with $l_{i,j}$ since v_i or v_j may be inside $I_{p,q}$.



(a) Due to RTS (b) Due to CTS

Figure 1: Communication Restriction by RTS/CTS.

Physical Interference Model (PhIM): In this model, the signal-to-interference-and-noise ratio (SINR) is used to describe the aggregate interference in the network. The transmission from node v_i is successfully received at node v_j if and only if the SINR is at least the minimum SINR threshold required by node v_j.

In this paper, we mainly focus on link scheduling for the fPrIM model and the RTS/CTS model. Notice that these two models are different. For example, in Figure 1(a), links BA and CD can be assigned the same channel in the protocol interference model, but not in the RTS/CTS model. Similar statement holds for links AB and DC in Figure 1(b).

2.2 Problem Formulation

Assume that the communication links in the wireless network are predetermined, either by some existing routing protocol as AODV, DSR or can be predicted from the existing routes. Given a communication graph $G = (V, E)$, we use the *conflict graph* (*e.g.*, [15]) F_G to represent the interference in G. Each vertex (denoted by $l_{i,j}$) of F_G corresponds to a directed link (v_i, v_j) in the communication graph G. There is an *edge* between vertex $l_{i,j}$ and vertex $l_{p,q}$ in F_G if and only if $l_{i,j}$ conflicts with $l_{p,q}$ due to interference. Recall that whether two links conflict depends on the interference model used underneath, *e.g.*, protocol interference model or RTS/CTS model. Thus, for a given communication graph G, the interference graph F_G may be different. To avoid the confusion, we use F_G^P to denote the interference graph under the protocol interference model and F_G^{D2} to denote interference graph under RTS/CTS model.

Our objective is to give each link $l \in G$ a transmission schedule $\mathcal{S}(l)$, which is the list of time slots it could send packets such that the schedule is interference-free and the overall throughout of the network is maximized. Let $X_{e,t} \in \{0,1\}$ be the indicator variable which is 1 iff e will transmit at time t. We will focus on periodic schedules in this paper. A schedule is periodic with period T if, for every link e and time slot t, $X_{e,t} = X_{e,t+i\cdot T}$ for any integer i. For a link e, let $I(e)$ denote the set of links e' that will cause interference if e and e' are scheduled at the same time slot. A schedule $\mathcal{S}$ is *interference-free* if $X_{e,t} + X_{e',t} \leq 1$ for any $e' \in I(e)$. In the graph theory terminology, the interference free link scheduling problem is essentially the *vertex coloring* of F_G.

When the traffic load of links are unknown, the objective of link scheduling is to find a scheduling with the minimum period. If we schedule all links within a period χ such that no two links in same time slot interfere with each other, then at least one packet can be delivered over each communication link in every χ time slots. Thus, $1/\chi$ is often used to estimate the *throughput* of the network based on this schedule. The second case is that the average traffic load $\ell(e)$ of each link is known in advance. We model this by assuming that each communication link e (vertex in the conflict graph) has a *weight* $w(e)$ specifying the minimum number of time slots it required in each period. Here $w(e) = \lceil T \cdot \frac{\ell(e)}{c(e)} \rceil$, where $c(e)$ is the capacity of link e and T is a given period for a schedule. Our main focus in this paper is how to schedule the communication links in an interference-free manner such that the throughput of the network is maximized, *i.e.*, with the smallest T.

There are a number of distinctions of the model used here with the models used in previous study: (1) We assume that each wireless node has an interference range, which may be different from its transmission range; (2) We do not require the same transmission range (also same interference range) for every wireless node; (3) We do not require the communication graph to be complete, *i.e.*, some communication links may not exist due to barriers or may be not used by routing selection.

Notice that for simplicity we assume that there is only a single-channel in the network. All our results can be easily extended to the case when multiple channels are available as in [1]. If nodes has a pre-assigned channels for each link, then the link scheduling with multiple channels is just the simple union of a set of schedulings, where each scheduling is for all links using the same channel. However, we agree that the static assignment of correct channels to appropriate links is a bigger factor in determining the performance. If links can dynamically switch channels, then our greedy algorithms will find the channel with the smallest available time slot for each link to be scheduled and the same performances hold.

3. CENTRALIZED SCHEDULING

In this section, we will propose centralized algorithms for link scheduling under different interference models. The performances of centralized algorithms will then be used as a certain benchmark to evaluate the performances of our distributed algorithms.

3.1 Scheduling under RTS/CTS Model

A number of centralized algorithms for link scheduling have been proposed in the literature, *e.g.*, [1, 21]. A common approach is to assign each link the best possible channels (smallest time slots here) by greedy. The difference between them is the processing order of links: [21] processes links with smaller lengths first while [1] processes links in an arbitrary order (since it uses UDG graph models for both communication and interference). Our centralized algorithm is will process links in a special order as in [14]. The basic idea is to first sort links as follows: every time we pick a link, say l, from the remaining graph that has the smallest number of interfered links in the remaining graph and then remove l from this graph; repeat this till the graph becomes empty. We then assign time slots to links in the reverse order of picked links using the smallest time slot available (not used by interfering links). In summary, a link e with larger $I(e)$ will be more likely processed earlier by our algorithm.

Algorithm 1 Centralized Scheduling under RTS/CTS Model

Input: A communication graph $G = (V, E)$ of m links.
Output: An interference-free link scheduling.

1: Construct the conflict graph F_G^{D2} and let graph $G' = F_G^{D2}$.
2: **while** G' is not empty **do**
3: Find the vertex with the *smallest* total degree in G' and remove this vertex from G' and all its incident edges. Let l_k denote the $(m-k+1)$th vertex removed, and the degree of l_k in graph G' just before it is removed be its δ-*degree*.
4: Process links from l_1 to l_m and assign to each l_k the smallest time slot not yet assigned to any of its neighbors in F_G^{D2}.

We first present some necessary definitions and properties needed to prove the performance of our algorithms. Given a communication link $l_{i,j}$, we define the *interference radius of link* $l_{i,j}$ as $r_{i,j} = \max\{r_i, r_j\}$. If $r_i > r_j$ or $r_i = r_j$ and ID of node v_i is larger than the ID of node v_j, then v_i is called the *head* (denoted as $h_{i,j}$) of link (v_i, v_j) and v_j is the *tail* (denoted as $t_{i,j}$) of this link. Notice that here, the *head* of a link is not necessarily the sender of the directed communication link. Given a node v_k, we use $R(v_k, x)$ to denote the disk centered at v_k and with radius $x \cdot r_k$. A node v_k interferes a node v_i if node v_i is inside the interference region (*i.e.*, disk $R(v_k, 1)$) of node v_k. We say a link $l_{p,q}$ interferes a node v_k if either v_p or v_q interferes v_k. For a given node v_k, we use $N^{\geq}(v_k, \alpha)$ to denote the set of nodes satisfying that (1) each of their interference radius is at least r_k; (2) each of them interferes some nodes in $R(v_k, \alpha)$. Notice that a node from $N^{\geq}(v_k, \alpha)$ could be arbitrarily far away from node v_k. Similarly, for a link $l_{i,j}$, let $R(l_{i,j}, x)$ denote the union of two disks centered at v_i and v_j respectively with radius $x \cdot r_i$ and $x \cdot r_j$ respectively. Let $N^{\geq}(l_{i,j}, \alpha)$ denote the union of node sets $N^{\geq}(v_i, \alpha)$ and $N^{\geq}(v_j, \alpha)$. The following theorem estimates the local chromatic number based on node degree.

THEOREM 1. *For a given node v_k and any node set $V_k \subseteq N^{\geq}(v_k, \alpha)$ with constant α, there exists a subset V'_k of V_k with cardinality $|V_k|/C_\alpha$ such that each node interferes with each other, where $C_\alpha \leq (6\alpha+1)^2 + 11$.*

PROOF. We consider a partition of V_k: the nodes in and outside region $R(v_k, 3\alpha)$, denoted by V_k^1 and V_k^2 respectively.

First, we consider the node set V_k^1. Using a simple area argument, there are at most $\frac{\pi((3\alpha+\frac{1}{2})r_k)^2}{\pi(\frac{1}{2}r_k)^2} = (6\alpha+1)^2$ disks with radius $\frac{r_k}{2}$ can be placed inside the disk $R(v_k, 3\alpha)$. Thus, there exists a node set in V_k^1 with size at least $|V_k^1|/(6\alpha+1)^2$ such that each node in the set interferes with each other.

(a) Divide the space into 11 cones (b) Two nodes interfere in same cone

Figure 2: Illustration of the partition of the region.

Second, we consider the node set V_k^2. We divide the whole space into 11 equal cones using 11 rays from v_k as shown Figure 2(a). If v_a and v_b are in the same cone, then $\angle v_a v_k v_b < 33°$. Let $d_{a,b} = \|v_a - v_b\|$. Since $v_a \in N^{\geq}(v_k, \alpha)$, v_a interfere with some nodes in $R(v_k, \alpha)$, $d_{a,k} \leq r_a + \alpha \cdot r_k$. Similarly, $d_{b,k} \leq r_b + \alpha \cdot r_k$. Thus, $\max\{d_{a,k}, d_{b,k}\} \leq \max\{r_a, r_b\} + \alpha \cdot r_k$. On the other hand, since both v_a and v_b are outside $R(v_k, 3\alpha)$, $\min\{d_{a,k}, d_{b,k}\} \geq 3\alpha \cdot r_k$. As shown in Figure 2 (b), for v_a and v_b,

$$\begin{aligned} d_{a,b}^2 &< d_{a,k}^2 + d_{b,k}^2 - 2\cos(33°) \cdot d_{a,k} \cdot d_{b,k} \\ &= \max\{d_{a,k}, d_{b,k}\}^2 + \min\{d_{a,k}, d_{b,k}\}^2 - \\ &\quad \frac{5}{3}\max\{d_{a,k}, d_{b,k}\} \cdot \min\{d_{a,k}, d_{b,k}\} \\ &\leq \max\{d_{a,k}, d_{b,k}\}\left[\max\{d_{a,k}, d_{b,k}\} - \frac{2}{3}\min\{d_{a,k}, d_{b,k}\}\right] \\ &\leq (\max\{r_a, r_b\} + \alpha \cdot r_k) \cdot [\max\{r_a, r_b\} + \alpha \cdot r_k - 2\alpha \cdot r_k] \\ &\leq \max\{r_a, r_b\}^2 - \alpha^2 \cdot r_k^2 < \max\{r_a, r_b\}^2. \end{aligned}$$

The transition between the second and third inequalities is because $\max\{d_{a,k}, d_{b,k}\} \leq \max\{r_a, r_b\} + \alpha \cdot r_k$ and $\min\{d_{a,k}, d_{b,k}\} \geq 3\alpha \cdot r_k$. Thus, v_a interferes with v_b. Therefore, each pair of nodes in the same cone interfere with each other. This proves that there exists a node set in V_k^2 with size at least $|V_k^2|/11$ such that the nodes in the set interfere with each other.

Consequently, there exists a node set with size at least

$$\max\{|V_k^1|/(6\alpha+1)^2, |V_k^2|/11\} \geq \frac{|V_k^1| + |V_k^2|}{(6\alpha+1)^2 + 11} = \frac{|V_k|}{C_\alpha}$$

such that all nodes in the set interfere with each other. Here, $C_\alpha \leq (6\alpha+1)^2 + 11$, and we call it the α-hop interference number. Notice that $(6\alpha+1)^2 + 11$ is an upper bound on C_α and it can be improved by using a more tight analysis. $\square$

Table 1: Summary of Main Notations

Term	Definition
v_i	a wireless node from $V = \{v_1, v_2, \cdots, v_n\}$
$l_{i,j}$ or (v_i, v_j)	edge/link between v_i and v_j
t_i and r_i	transmission range and interference range of v_i
γ_i and γ	$\gamma_i = r_i/t_i$, γ is the maximum ratio for all v_i
$I_{i,j}$,$h_{i,j}$,$t_{i,j}$	interference region/head/tail of link $l_{i,j}$
$r_{i,j}$	interference radius of $l_{i,j}$, $\max\{r_i, r_j\}$
$X_{e,t}$	indicator variable whether e transmits at time t
F_G^{D2}	interference graph under RTS/CTS model
F_G^P	interference graph under fPrIM model
$\delta(F_G^X)$	maximum δ-degree in the interference graph
$\Delta(F_G^X)$	maximum degree in the interference graph
$R(v_k, x)$	the disk centered at v_k and with radius $x \cdot r_k$
$R(l_{i,j}, x)$	the union of two disks centered at v_i and v_j respectively with radius $x \cdot r_i$ and $x \cdot r_j$
$N^{\geq}(v_k, \alpha)$	the set of nodes who interferes some nodes in $R(v_k, \alpha)$ and has interference radius at least r_k
$N^{\geq}(l_{i,j}, \alpha)$	the union of node sets $N^{\geq}(v_i, \alpha)$ and $N^{\geq}(v_j, \alpha)$
$I^{\geq}(e)$	the set of links with larger radius than e and interfering with e under RTS/CTS model
$d_{i,j}^{in/out}(G)$	incoming/outgoing degree of vertex $l_{i,j}$ in G
$\Delta^{in/out}(G)$	maximum incoming/outgoing degree of G
$d_{i,j}^{\geq}(F_G^{D2})$	number of adjacent vertices that precede $l_{i,j}$
$\phi(F_G^{D2})$	$\max_{l_{i,j}} d_{i,j}^{\geq}(F_G^{D2})$
H_i	all links that contain v_i as the head
M_i,M_i^+,M_i^-	all links that $\notin H_i$ and interfere with H_i; all links in M_i that precede every link in H_i; $M_i - M_i^+$
ψ	ratio between max and min interference ranges
$N^{\geq}(v_k, \alpha, \beta)$	the set of nodes who interferes some nodes in $R(v_k, \alpha)$ and has interference radius at least $\frac{r_k}{\beta}$
$N^{\geq}(l_{i,j}, \alpha, \beta)$	the union of $N^{\geq}(v_i, \alpha, \beta)$ and $N^{\geq}(v_j, \alpha, \beta)$
$\Delta(\alpha, \beta)$	$\max_{l_{i,j}} \lvert N^{\geq}(l_{i,j}, \alpha, \beta)\rvert$
$\chi(F_G^X)$	optimal number of colors for graph F_G^X
$M_{i,j}^{in/out}$	all incomging/outgoing links from $l_{i,j}$
$w(e)$,$\ell(e)$,$c(e)$	weight, traffic load and capacity of link e

Notice that Theorem 1 works for the interference on nodes only. For a link $e = l_{i,j}$, let $I^{\geq}(e)$ be the links e' interfering with e under RTS/CTS model and whose radius is not smaller than e. Following theorem shows a counterpart that works for links also.

THEOREM 2. *For a given link $e = l_{i,j}$, at least $|I^{\geq}(e)|/(2C_1)$ time slots are needed to schedule all links in $I^{\geq}(e)$.*

PROOF. For each link $l_{p,q} \in I^{\geq}(e)$, without loss of generality, we assume that $r_p \geq r_q$. Recall that $e' = l_{p,q}$ and e interfere by definition. Following we discuss by cases.

Case 1: The interference region of v_p covers either v_i or v_j.

Case 2: The interference region of node v_p can neither cover v_i nor v_j, and v_q is *outside* the union $R(l_{ij}, 1)$ of interference region of v_i and v_j. Clearly, in this case v_p must also be outside of $R(l_{ij}, 1)$. Since e and e' interfere, it must be that the interference region of v_q covers either v_i or v_j.

Case 3: The interference region of node v_p can neither cover v_i nor v_j, and v_q is *inside* the union $R(l_{ij}, 1)$ of interference region of v_i and v_j. Then v_p will "interfere" a dummy node v_q.

In summary, we conclude that at least one end node of $l_{p,q}$ interferes with some nodes in region $R(l_{i,j}, 1)$, *i.e.*, the head of $l_{p,q}$ is in $N^{\geq}(l_{i,j}, 1)$. Recall that $N^{\geq}(l_{i,j}, 1) = N^{\geq}(v_i, 1) \bigcup N^{\geq}(v_j, 1)$. The head of $l_{p,q}$ is either in $N^{\geq}(v_i, 1)$ or $N^{\geq}(v_j, 1)$. Without loss of generality, we assume that at least $|I^{\geq}(e)|/2$ heads of the links in $I^{\geq}(e)$ are in $N^{\geq}(v_i, 1)$. From Theorem 1, there are at least $|I^{\geq}(e)|/(2C_1)$ heads that interfere with each other. Thus, there are at least $|I^{\geq}(e)|/(2C_1)$ links in $I^{\geq}(e)$ that interfere with each other. This finishes the proof. □

Consequently, we have the following necessary condition for any interference-free link scheduling under RTS/CTS model:

LEMMA 3. *For any time slot τ, any valid RTS/CTS interference-free link scheduling $\mathcal{S}$ must satisfy that*

$$X_{e,\tau} + \sum_{e' \in I^{\geq}(e)} X_{e',\tau} \leq 2C_1.$$

Notice that above theorems hold for any multi-hop wireless networks in which both the transmission range and interference range could be heterogenous and some links could be missing due to various reasons. If the interference range is homogenous, then the constant C_α could be improved.

Let $\delta(F_G^{D2})$ be the *maximum* δ-degree of all links l_k in the Step 2-3 of Algorithm 1. We now prove that Algorithm 1 has the following performance guarantee.

THEOREM 4. *Under RTS/CTS model, Algorithm 1 needs at most $2C_1 \cdot \delta_{opt}$ time-slots for all links without interference, where δ_{opt} is the minimum schedule period T.*

PROOF. Let H be the vertex induced subgraph of F_G^{D2} such that each vertex in H has degree at least $\delta(F_G^{D2})$. The existence of H is straightforward from the definition of $\delta(G)$. Without loss of generality, let $l_{i,j}$ be the vertex in H with the smallest interference range. From Theorem 2, there exists a clique of size at least $\frac{\delta(F_G^{D2})+1}{2C_1}$ in F_G^{D2}. The optimal solution thus needs $\geq \frac{\delta(F_G^{D2})+1}{2C_1}$ colors. Algorithm 1 uses $\leq \delta(F_G^{D2}) + 1$ colors. This finishes our proof. □

3.2 Scheduling under fPrIM Model

Kumar *et al.* [21] studied the scheduling under a different protocol interference model (with parameter δ): where a transmission by a node v_i is successfully received by a node v_j iff $\|v_k - v_j\| \geq (1 + \delta)\|v_i - v_j\|$ for any node $v_k \neq v_i$. This needs every node to dynamically change its transmission power based on receiving node. Recall that in this paper, we assume that any node will have a fixed transmission power. It is not difficult to design network examples where the methods (processing links in the order of decreasing length) developed in [21] will not work under our model.

Under RTS/CTS model, we essentially showed that the optimal color assignment needs at least $\delta(F_G^{D2})$ colors. Note that when the graph is modelled by UDG, $\delta(F_G^{D2})$ is essentially $\Delta(F_G^{D2})$, where $\Delta(F_G^{D2})$ is the maximum degree of the conflict graph F_G^{D2}. Thus, almost any greedy based coloring method (using at most $\Delta(F_G^{D2}) + 1$ colors) has a constant approximation ratio. Several previous literatures claimed the same result (that the optimal coloring needs $\Theta(\Delta(F_G^P))$ colors) under the fPrIM model and proposed some algorithms to color the communication graph G using $O(\Delta(F_G^P))$ colors, where $\Delta(F_G^P)$ is the maximum degree of the conflict graph F_G^P under fPrIM model. We can also define $\delta(F_G^P)$ as the maximum δ-degree of the F_G^P which can be computed by applying Step 2-3 of Algorithm 1 on F_G^P. However, as we will show later, there are examples of communication graphs whose optimal coloring needs constant colors, while, on the other hand, both $\Delta(F_G^P)$ and $\delta(F_G^P)$ are $O(n^{1-\epsilon})$ for any $0 \leq \epsilon < 1$ if all nodes have the same transmission range and $t_i = r_i = r$. This shows that any greedy algorithm that uses $\Theta(\Delta(F_G^P))$ or even $\Theta(\delta(F_G^P))$ colors could be very bad compared to the optimal solution.

We now describe such an example as in Figure 3. Here all nodes have same transmission range and interference range r. The links formed several groups such that all links in each group are parallel and each link has length r. The groups are placed in a cyclic manner such that any sender of one group interferes with all receivers in the previous group and does not interfere with any other receivers in other groups. The number of links in each group is $n^{1-\epsilon}$ and there are n^{ϵ} groups. Obviously, in the conflict graph F_G^P, the degree of each vertex (corresponding to a physical link) is $n^{1-\epsilon}$. Thus, $\Delta(F_G^P) = \delta(F_G^P) = n^{1-\epsilon}$. On the other hand, we can use at most 3 colors to color all the links without conflict: we color groups in clockwise order, and all links in the same group are assigned the same color that is the smallest available.

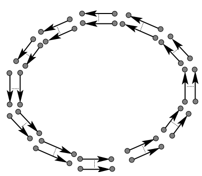


Figure 3: Bad example for simple greedy

The above example shows that it is unclear whether Algorithm 1 can find a scheduling that approximates the optimal solution when the interference range equals the transmission range (the proof of Theorem 4 does not extend to this scenario). Fortunately, the ratio of the interference range over the transmission range is usually around 2 in practice. Next, we utilize this property to design an efficient link scheduling with a constant approximation ratio.

Given any two nodes $l_{i,j}$ and $l_{p,q}$ in conflict graph F_G^P such that v_j and v_q are receivers, if $l_{i,j}$ and $l_{p,q}$ interfere with each other, then it is possible that (1) v_i interferes v_q, or (2) v_p interferes v_j, (3) or both. If v_p interferes v_j, then we treat the link between $l_{i,j}$ and $l_{p,q}$ as an *incoming link* for $l_{i,j}$. Similarly, if v_i interferes v_q, we treat the link as an *outgoing link* for $l_{i,j}$. Let $d_{i,j}^{in}(F_G^P)$ and $d_{i,j}^{out}(F_G^P)$ be the incoming and outgoing degree of $l_{i,j}$ in the conflict graph F_G^P respectively. The number of incoming links of a vertex in F_G^P is its incoming degree, and the number of outgoing links are its outgoing degree. Similarly, we define $\Delta^{in}(F_G^P)$ and $\Delta^{out}(F_G^P)$ as the maximum incoming and outgoing degree in graph F_G^P respectively. When $\gamma_i > 1$ for each node v_i, we will show that the optimal coloring needs at least $\Theta(\Delta^{in}(F_G^P))$ colors, where the hidden constant depending on $\min_i \gamma_i$ (which is typically 2 in practice).

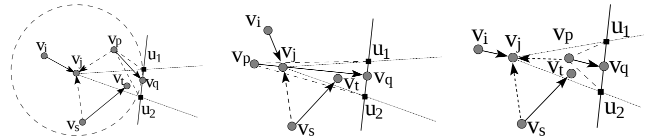
LEMMA 5. *Consider any communication link $l_{i,j}$, where v_j is the receiver. Consider two links $l_{p,q}$ and $l_{s,t}$ that are $l_{i,j}$'s incoming links in conflict graph F_G^P, where v_q and v_t are the receivers. If $\angle v_q v_j v_t \le \arcsin \frac{\gamma-1}{2\gamma}$, then link $l_{p,q}$ interferes with link $l_{s,t}$.*

PROOF. Draw two rays $v_j v_a$, $v_j v_b$ emanated from node v_j such that $\angle v_a v_j v_b = \arcsin \frac{\gamma-1}{2\gamma}$ and v_q, v_t are in the cone as shown in Figure 4(a). Without loss of generality, we assume that $\|v_j - v_q\| \ge \|v_j - v_t\|$. Draw a circle $\mathcal{C}$ centered at v_j with radius $\|v_j - v_q\|$. Let $u_1 u_2$ be the line passing v_q that is tangent to circle $\mathcal{C}$ and u_1, u_2 are the intersections of this line with line $v_j v_a$, $v_j v_b$ respectively. Since $\angle u_1 v_j v_q \le \arcsin \frac{\gamma-1}{2\gamma}$, we have

$$\|u_1 - v_q\| \le \|v_j - v_q\| \cdot \frac{\gamma - 1}{2\gamma} \le 2r_p \cdot \frac{\gamma-1}{2\gamma} = r_p \cdot \frac{\gamma-1}{\gamma}.$$

Thus, $\|v_p - u_1\| \le \|v_p - v_q\| + \|u_1 - v_q\| \le r_p \cdot \frac{1}{\gamma} + r_p \cdot \frac{\gamma-1}{\gamma} = r_p$. Similarly, $\|v_p - u_2\| \le r_p$. Following we prove that node v_p interferes with v_t by cases.

Case 1**:** $v_p u_1 u_2 v_j$ is a convex quadrangle as shown in Figure 4(a). In this case, v_t is either inside triangle $v_p v_j u_2$ or triangle



(a) Case 1 (b) Case 2 (c) Case 3

Figure 4: Links in a small neighborhood will interfere with each other in protocol interference model.

$v_p u_1 u_2$. Since both $\|v_p - u_1\|$, $\|v_p - u_2\|$ and $\|v_p - v_j\|$ are not greater than r_p, we have $\|v_p - v_t\| \le r_p$.

Case 2**:** v_j is inside $\triangle u_1 u_2 v_p$ as shown in Figure 4(b). In this case, v_t is inside triangle $\triangle u_1 u_2 v_p$. Then it is easy to show that $\|v_p - v_t\| \le \max\{\|v_p - u_1\|, \|v_p - u_2\|\} \le r_p$.

Case 3**:** v_p is inside $\triangle u_1 u_2 v_j$ as shown in Figure 4(c). In this case, v_t is inside one of the three triangles: $\triangle u_1 u_2 v_p$, $\triangle u_1 v_j v_p$, $\triangle u_2 v_j v_p$. Similarly, we have $\|v_p - v_t\| \le r_p$.

Obviously, the above three cases covers all possible situations. This proves that link $l_{p,q}$ interferes with $l_{s,t}$. □

Similar to Lemma 3, we have the following necessary condition for interference-free link scheduling under fPrIM model.

LEMMA 6. *For any time slot τ, any valid interference-free link scheduling $\mathcal{S}$ under protocol interference model must satisfy that*

$$X_{e,\tau} + \sum_{e' \in I^{in}(e)} X_{e',\tau} \le \lceil \frac{2\pi}{\arcsin \frac{\gamma-1}{2\gamma}} \rceil,$$

where $I^{in}(e)$ is the set of incoming links of e that interfere e.

This is because that for all incoming neighboring links of link e, Lemma 5 implies that there are at most $\lceil \frac{2\pi}{\arcsin \frac{\gamma-1}{2\gamma}} \rceil$ links that can be scheduled at any same time slot. We then present our main theorem about the optimum coloring for fPrIM model with $\gamma_i > 1$.

THEOREM 7. *Optimal vertex coloring for conflict graph F_G^P needs $\Theta(\Delta^{in}(F_G^P))$ colors if $\min_i \gamma_i$ is some constant > 1.*

PROOF. For any link $l_{i,j}$ such that v_j is the receiver, we partition the space using b equal-sized cones apexed at node v_j, where $b = \lceil \frac{2\pi}{\arcsin \frac{\gamma-1}{2\gamma}} \rceil$. From the Pigeon hole principle, $l_{i,j}$ has at least $d_{i,j}^{in}(F_G^P)/b$ links whose receivers are in the same cone. From Lemma 5, all links in the same cone interfere with each other. Thus, $l_{i,j}$ has at least $d_{i,j}^{in}(F_G^P)/b$ in-coming links such that they interfere with each other. It implies that any valid coloring will use at least $d_{i,j}^{in}(F_G^P)/b$ among the incoming neighbors of link $l_{i,j}$. Thus, the optimal coloring needs at least $\Delta^{in}(F_G^P)/b + 1$ colors. □

Note that $\Delta(F_G^P)$ could be arbitrary larger than $\Delta^{in}(F_G^P)$. Thus, simple greedy algorithm using $\Delta(F_G^P)$ colors does not work, *e.g.*, the algorithm proposed in [1] for UDG networking model. It is known that the optimal coloring can be obtained by using greedy approach on a certain ordering of vertices in F_G^P. Next, with a careful selection of link ordering, we present our centralized scheduling method (Algorithm 2) that needs at most $2 \cdot \Delta^{in}(F_G^P) + 1$ colors.

THEOREM 8. *Algorithm 2 uses at most $2 \cdot \Delta^{in}(F_G^P) + 1$ colors.*

PROOF. The key observation is that in any directed graph, the sum of all vertices' incoming degree equals the sum of outgoing degree. For the link $l_{i,j}$ with the largest $d_{i,j}^{in}(G') - d_{i,j}^{out}(G')$ in G', we must have $d_{i,j}^{in}(G') \ge d_{i,j}^{out}(G')$. Thus, when we assign color

Algorithm 2 Centralized Scheduling under Protocol-Interference

Input: A communication graph $G = (V, E)$ of m links.
Output: An interference-free link scheduling.

Construct the conflict graph F_G^{D2} and let graph $G' = F_G^{D2}$.
while G' is not empty **do**
  Find the link $l_{i,j}$ with the *largest* $d_{i,j}^{in}(G') - d_{i,j}^{out}(G')$ in G' and remove this vertex from G' and all its incident edges. Let l_k denote the kth vertex removed.
Process the sequences of links $l_{i,j}$ from l_m to l_1. Assign each link l_k the smallest time slot not yet assigned to any of its neighbors in F_G^P.

(or time-slot) for the link $l_{i,j}$, the subgraph induced by all the links that have already been processed is exactly the subgraph G' right before vertex $l_{i,j}$ was removed in the **while** loop of Algorithm 2. Therefore, there are at most $2 \cdot d_{i,j}^{in}(G')$ adjacent neighbors of $l_{i,j}$ in F_G^P that have already been processed. In other words, the smallest time-slot assigned to $l_{i,j}$ is at most $2 \cdot d_{i,j}^{in}(G') + 1$, which is at most $2 \cdot d_{i,j}^{in}(F_G^P) + 1$. This proves that we need at most $2 \cdot \Delta^{in}(F_G^P) + 1$ time-slots for an interference-free schedule. $\square$

4. DISTRIBUTED ALGORITHMS

In a wireless network, centralized algorithm may not be possible and even if possible, due to the dynamic features of wireless networks, it is inefficient to update the coloring using a centralized algorithm. Thus, in this section, we design efficient distributed algorithms to get a valid coloring with good performance guarantee.

4.1 Algorithm For RTS/CTS Model

In literatures, several distributed algorithms have been proposed for the vertex coloring. The first solution is to simply apply a distributed vertex coloring on the conflict graph F_G^{D2}. Recall that all previous distributed algorithms work for the general graph. By taking advantage of special properties of conflict graph defined here, we are able to obtain a deterministic distributed coloring algorithm that colors the links with $O(\Delta(F_G^{D2}))$ colors in almost constant time when the interference ranges are homogenous. On the other hand, as shown in our centralized algorithm, the optimal color is $\Theta(\delta(F_G^{D2}))$ which could be much smaller than $\Delta(F_G^{D2})$ when interference ranges are heterogenous. Thus, simply applying a coloring algorithm with ratio $\Theta(\Delta(F_G^{D2}))$ may not achieve a good performance. The first instinct is to design a distributed version of Algorithm 1. However, finding the node with the global maximum degree iteratively does not seem promising for distributed algorithm. Thus, we need to find some lower bound for the optimal color other than $O(\delta(F_G^{D2}))$.

Given two nodes v_i and v_j, we say that v_i *precedes* v_j if and only if $r_i > r_j$ or $r_i = r_j$ and $i > j$. Given a pair of links $l_{i,j}$ and $l_{p,q}$ with different heads $h_{i,j} \neq h_{p,q}$, we say that $l_{i,j}$ *precedes* $l_{p,q}$ if $r_{i,j} > r_{p,q}$ or $r_{i,j} = r_{p,q}$ and $h_{i,j} > h_{p,q}$. Recall that $r_{i,j} = \max\{r_i, r_j\}$. We also say that the corresponding vertex $l_{i,j}$ precedes $l_{p,q}$ in the conflict graph in this case. For a vertex $l_{i,j}$ in graph F_G^{D2}, let $d_{i,j}^{\geq}(F_G^{D2})$ be the number of adjacent vertices that precede $l_{i,j}$, which is called *efficient degree* of $l_{i,j}$. From Theorem 2, there are at least $d_{i,j}^{\geq}(F_G^{D2})/(2C_1)$ vertices adjacent to and preceding $l_{i,j}$ that form a clique in which each vertex (*i.e.*, the corresponding link in the communication graph) interferes with each other. Let $\phi(F_G^{D2}) = \max_{l_{i,j}} d_{i,j}^{\geq}(F_G^{D2})$, then Theorem 2 shows that optimal coloring algorithm needs at least $\phi(F_G^{D2})/(2C_1)$ colors. Thus, finding a coloring algorithm using at most $\Theta(\phi(F_G^{D2}))$ colors is a constant-ratio approximation algorithm. Unlike the centralized Algorithm 1 in which the lower bound of $\delta(F_G^{D2})$ could not be found by using only local information, the lower bound of $\phi(F_G^{D2})$ could be easily obtained by any link $l_{i,j}$ by simply counting the number of interfering links that precede itself, *i.e.*, with larger link interference radius. Algorithm 3 presents our distributed coloring method that uses at most $\phi(F_G^{D2})$ colors.

Algorithm 3 Distributed Coloring Algorithm for RTS/CTS Model

Input: A communication graph $G = (V, E)$.
Output: A valid coloring of all links.

Each node v_i collects all communication links, say H_i, that contain v_i as the head, *i.e.*, all links $l_{i,j}$ with $r_i \geq r_j$.
Each node v_i collects all communication links, denoted by M_i, that are not in H_i and interfere with some links H_i.
Node v_i finds M_i^+, which is the subset of links in M_i that precedes *every* link in H_i and let $M_i^- = M_i - M_i^+$.
Node v_i sets all links in M_i^+ as uncolored.
while some links in M_i^+ are uncolored **do**
  Node v_i listens messages from other nodes.
  if v_i receives a message Color(p, q, k) **then**
    Node v_i marks $l_{p,q}$ with color ID k if $l_{p,q}$ is in M_i^+.
for each node v_j in H_i **do**
  Find the color with minimum color ID, say k, that is not used by any link that is conflicted with $l_{i,j}$. Color link $l_{i,j}$ with color ID k.
  Sends the message Color(i, j, k) to all heads of the links adjacent to $l_{i,j}$ in M_i^-.

THEOREM 9. *Algorithm 3 computes a valid coloring using at most $\phi(F_G^{D2})$ colors, which is asymptotically optimal.*

PROOF. First, we show that the algorithm does terminate. Since it is straightforward that the number of nodes in H_i is bounded by $\phi(F_G^{D2})$, the **for** loop terminates in $O(n)$ iterations. Thus, the maximum time needed for all other processes other than **while** loop is bounded by a finite time T and our main focus is to show that the **while** loop does terminate for any node v_i. Let $(v_{\sigma_1}, v_{\sigma_2}, \ldots, v_{\sigma_n})$ be the sorted list of nodes in the decreasing order of their interference range. Thus, v_{σ_i} precedes v_{σ_j} if and only if $i < j$. Since v_{σ_1} precedes every other nodes, $M_{\sigma_1}^+$ is empty and v_{σ_1} colors all links that are adjacent to v_{σ_1} in time T. Now consider the node v_{σ_2} and $M_{\sigma_2}^+$. If $l_{p,q} \in M_{\sigma_2}^+$, then either v_p or v_q is v_{σ_1}. Thus, all links in $M_{\sigma_2}^+$ are colored. Therefore, all links that are adjacent to v_{σ_2} are colored before time $2T$. Similarly, all links that are adjacent to v_{σ_j} are colored before time $j \cdot T$. Thus, all links are colored in time $n \cdot T$. It is straightforward to show that, by assuming color one link takes a unit time, the running time of this algorithm is at most m, where m is the number of directed communication links.

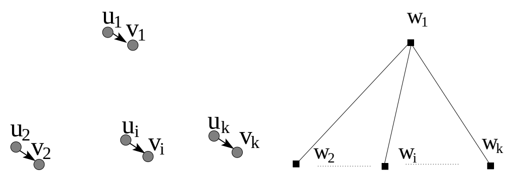
Second, we show that the computed coloring is valid, *i.e.*, no two conflict links have the same color. Consider conflict links $l_{i,j}$ and $l_{p,q}$, following we discuss by cases.

Case 1: $l_{i,j}$ and $l_{p,q}$ have the same head. Without loss of generality, we assume that $v_i = v_p$ is the head of the links. Thus, both $l_{i,j}$ and $l_{p,q}$ are in H_i. Therefore, $l_{i,j}$ and $l_{p,q}$ have different colors.

Case 2: $l_{i,j}$ and $l_{p,q}$ have different heads. Then, without loss of generality, we can assume that $h_{i,j} = i$, $h_{p,q} = p$ and v_i precedes v_p. Since $l_{i,j} \in M_p^+$, $l_{i,j}$ is colored before M_p^+ becomes empty. Thus, $l_{p,q}$ is colored after $l_{i,j}$ is. Therefore, when v_p colors $l_{p,q}$, it uses a color that is different from the color of $l_{i,j}$ based on our algorithm.

Third, it is straightforward that Algorithm 3 uses at most $\phi(F_G^{D2})$ colors, *i.e.*, it has a constant approximation ratio. $\square$

Notice that in Algorithm 3, we start to color a link after all interfering links preceding it are colored. Thus, in the worst case, it may take time $O(n)$ to color all the links, where n is the number of nodes in the network. Here we assume that in one time unit, a node can color all its incident links. Comparing with previous poly-logarithmic time distributed coloring algorithms that color the graph using $\Delta(F_G^{D2})$ colors, Algorithm 3 may take longer time. However, following example shows that $\Delta(F_G^{D2})$ could be as large



(a) The Original network (b) The Conflict Graph

Figure 5: Δ could be $\Theta(n)$ of number of colors used by Alg. 3.

as $O(n)$ times of the color used by Algorithm 3, where n is the number of the nodes in original network. In Figure 5(a), there are k pairs of transmission links $u_1v_1, \ldots, u_nv_n$. Nodes u_1, v_1 have interference range 1 and all other nodes have interference range ϵ, where ϵ is a small positive constant such that node u_i does not interfere v_j for $i, j > 1$. The corresponding conflict graph is shown in Figure 5(b). It is not difficult to see that we only need two colors while the degree of $l_{1,1}$ is $n-1$. In other words, compared with previous poly-logarithmic time methods with $\Omega(n)$ approximation ratios, our method has a constant approximation ratio using larger worst-case running time.

4.2 Faster Algorithm For RTS/CTS Model

Although Algorithm 3 computes a coloring that is at most constant times of the optimal, it may need linear number of rounds to compute the coloring. In certain circumstances, we would prefer the distributed algorithms that run fast to the distributed algorithms that have good performance as long as the fast distributed algorithm does not perform much worse. Following we present another distributed algorithm that computes the coloring very fast with a good performance guarantee of $O(\log(\psi)+1)$, where ψ is the ratio between the maximum interference range over the minimum interference range among all nodes.

Algorithm 4 Fast Distributed Coloring Algorithm For RTS/CTS

Input: A communication graph $G = (V, E)$.
Output: A valid coloring of the communication graph.

```
Node v_i computes a subset, say H_i, of all communication links
    containing v_i such that link l_{i,j} ∈ H_i if and only if r_i > r_j.
while node v_i failed to obtain the channel do
    Node v_i monitors the channel and competes for the channel.
for each link l_{i,j} ∈ H_i do
    Color link l_{i,j} with the smallest color ID, say k, that is not
    used by any link that conflicts with l_{i,j}.
    Broadcasts the message Color(i, j, k) to each head of links
    that conflict with l_{i,j}.
```

Algorithm 4 assumes that there is certain competition based MAC layer (*e.g.,* 802.11 with RTS/CTS) available for a node to obtain the channel. We use this MAC mechanism to obtain a link scheduling that is efficient and interference free. Algorithm 4 is very simple and can be implemented without much additional computation on each node. However, the proof of the performance guarantee is not straightforward. To prove the main theorem, we need some notation in order to extend the Theorem 1 and Theorem 2. For a given node v_k, Let $N^{\geq}(v_k, \alpha, \beta)$ be a node set composed of the nodes satisfying that (1) each of their interference radius is at least $\frac{r_k}{\beta}$; (2) each of them interferes some nodes in $R(v_k, \alpha)$. Let $N^{\geq}(l_{i,j}, \alpha, \beta)$ be the union of $N^{\geq}(v_i, \alpha, \beta)$ and $N^{\geq}(v_j, \alpha, \beta)$. The proofs of the following Lemma 10 and 11 are similar to the proofs of Theorem 1 and 2 respectively and thus are omitted here.

LEMMA 10. *For any node v_k and any set $V_k \subseteq N^{\geq}(v_k, \alpha, \beta)$, there exists a subset V_k' of V_k with cardinality at least $\lceil |V_k|/C_{\alpha,\beta} \rceil$ such that nodes in V_k' interfere with each other where $C_{\alpha,\beta} = (6\alpha\beta+1)^2 + 11$.*

LEMMA 11. *For any link $l_{i,j}$ and any set $V_{ij} \subseteq N^{\geq}(l_{i,j}, \alpha, \beta)$, there exists a subset V_{ij}' of V_{ij} with cardinality at leat $\lceil V_{ij}/(2C_{\alpha+1,\beta}) \rceil$ such that links in V_{ij}' interfere with each other.*

Let $\Delta(\alpha, \beta) = \max_{l_{i,j}} |N^{\geq}(l_{i,j}, \alpha, \beta)|$ and $\chi(F_G^{D2})$ be the optimal number of colors. Based on Lemma 11, the following theorem is straightforward, for any fixed α, β,

THEOREM 12. $\chi(F_G^{D2}) \geq \lceil \Delta(\alpha, \beta)/(2C_{\alpha+1,\beta}) \rceil$.

THEOREM 13. *Algorithm 4 computes a coloring that is at most $O(\log(\psi)+1)$ times of optimum $\chi(F_G^{D2})$.*

PROOF. Without loss of generality, let link $l_{i,j}$ be the link that has the maximum color ID, say **g**. To prove the theorem, we will show that $\mathbf{g} \leq 2C_{1,2} \cdot (\log(\psi)+1) \cdot \chi$. Following we prove it by contradiction and for the sake of contradiction, assume that $\mathbf{g} > 2C_{1,2} \cdot (\log(\psi)+1) \cdot \chi$.

We first argue that for any $0 \leq k \leq \log(\psi)$, there exists a link $l_{i^{(k)},j^{(k)}}$ such that $r_{i^{(k)},j^{(k)}} < r_{i,j}/2^k$ and its color ID is not smaller than $\mathbf{g} - 2C_{1,2} \cdot k \cdot \chi$. We prove this argument by induction on k. If $k = 0$, then the argument trivially holds. Assume for $k \leq p$, the argument holds. From Theorem 12, by letting $\alpha = 0$ and $\beta = 2$, $\chi \geq \Delta(0,2)/(2C_{1,2})$. In other words, the number of links, that interfere or are interfered by link $l_{i^{(p)},j^{(p)}}$ and whose radius is not smaller than $r_{i^{(p)},j^{(p)}}/2$, is at most $2C_{1,2} \cdot \chi$. Thus, there must exist a link $l_{i^{(p+1)},j^{(p+1)}}$ such that

1. $l_{i^{(p+1)},j^{(p+1)}}$ interferes or is interfered by $l_{i^{(p)},j^{(p)}}$;
2. $r_{i^{(p+1)},j^{(p+1)}} < r_{i,j}/2^{p+1}$; and
3. $l_{i^{(p+1)},j^{(p+1)}}$'s color ID is at least $\mathbf{g} - 2C_{1,2} \cdot (p+1) \cdot \chi$.

This finishes the induction.

Thus, let $k = \lfloor \log(\psi) \rfloor$, link $l_{i^{\lfloor \log(\psi) \rfloor},j^{\lfloor \log(\psi) \rfloor}}$ has the color ID not smaller than $\mathbf{g} - 2C_{1,2} \cdot \lfloor \log(\psi) \rfloor \cdot \chi$. This implies that $l_{i^{\lfloor \log(\psi) \rfloor},j^{\lfloor \log(\psi) \rfloor}}$ has at least $2C_{1,2} \cdot \chi + 1$ adjacent links. Since, $r_{i^{(\lfloor \log(\psi) \rfloor)},j^{(\lfloor \log(\psi) \rfloor)}} < r_{i,j}/2^{\lfloor \log(\psi) \rfloor)}$ and $r_{p,q} \geq r_{i,j}/2^{\log(\psi)}$, all links that interfere or are interfered by link $l_{i^{\lfloor \log(\psi) \rfloor},j^{\lfloor \log(\psi) \rfloor}}$ have interference radius at least $r^{i^{\lfloor \log(\psi) \rfloor},j^{\lfloor \log(\psi) \rfloor}}/2$. From Lemma 11, $\chi \geq \lceil \frac{2C_{1,2} \cdot \chi + 1}{2C_{1,2}} \rceil = \chi + 1$, which is a contradiction. Thus, $\mathbf{g} \leq 2C_{1,2} \cdot (\log(\psi)+1) \cdot \chi$. This finishes the proof. $\square$

4.3 Distributed Algorithm Under fPrIM Model

From Theorem 8, any coloring algorithm that uses $O(\Delta^{in}(F_G^P))$ colors under the fPrIM model has a constant approximation ratio. Here we give a distributed algorithm (Algorithm 5) that bears the similar idea of our centralized method (Algorithm 2).

Regarding the distributed method (Algorithm 5), we have:

THEOREM 14. *Algorithm 5 computes a valid coloring with at most $2 \cdot \Delta^{in}(F_G^P) + 1$ colors with $O(m)$ messages, where m is the number of communication links.*

Algorithm 5 Distributed Scheduling for fPrIM model

Input: A communication network $G = (V, E)$.
Output: A valid coloring of all links.

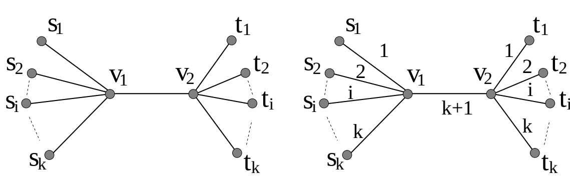
1: Assign each communication link a label WHITE.
2: The header of each communication link $l_{i,j}$ collects all incoming links and outgoing links, denoted by $M_{i,j}^{in}$ and $M_{i,j}^{out}$.
3: **while** link $l_{i,j}$ is WHITE **do**
4: Link $l_{i,j}$ monitors the channel.
5: If some link e in $M_{i,j}^{in} \bigcup M_{i,j}^{out}$ announces that it becomes GRAY with time-stamp k, link $l_{i,j}$ locally stores the label of link e as GRAY and the time stamp k.
6: **if** the number of WHITE links in $M_{i,j}^{in}$ is not smaller than the number of WHITE links in $M_{i,j}^{out}$ **then**
7: Link $l_{i,j}$ competes for the channel.
8: **if** Link $l_{i,j}$ obtains the channel **then**
9: Link $l_{i,j}$ labels itself GRAY with a time stamp $t + 1$ where t is the maximum time stamp of all GRAY links stored locally. Here $t = 0$ is no GRAY links are stored. Link $l_{i,j}$ send to all adjacent links in F_G^P the message that $l_{i,j}$ becomes GRAY with the time stamp $t+1$. Link $l_{i,j}$ makes a list of links $S_{i,j}$ composed of the current WHITE links in $M_{i,j}^{in} \bigcup M_{i,j}^{out}$.
10: **while** there exists some links in $S_{i,j}$ not colored **do**
11: Link $l_{i,j}$ listens to the announcement. If a link e' in $S_{i,j}$ announces its color, then link $l_{i,j}$ locally updates the status of e' as colored together with the color of e'.
12: Link $l_{i,j}$ colors itself using the smallest color available that will not produce any conflict with links in $S_{i,j}$. It then sends to all adjacent links in F_G^P without a color the message about its current color assigned.

PROOF. Notice that for each link $l_{i,j}$, it uses the smallest color that is not used by any links in $S_{i,j}$. Since the number of incoming links is not smaller than the outgoing links in $S_{i,j}$, link $l_{i,j}$ is colored with a color not greater than $2 \cdot d_{i,j}^{in}(F_G^P) + 1$. Thus, Algorithm 5 computes a valid coloring with at most $2 \cdot \Delta^{in}(F_G^P) + 1$ colors. Note that each link $l_{i,j}$ only announces twice in our distributed scheduling algorithm: when it becomes GRAY and when it is colored. Thus, the overall message complexity is $O(m)$. $\square$

5. WEIGHTED COLORING AND SCHEDULABLE FLOWS

5.1 Scheduling With Traffic Load

In TDMA system, the minimization of the number of colors is closely related to the maximization of the network throughput. One intrinsic assumption behind the idea of coloring is that each communication link has the same packet arrive rate, *i.e.*, the number of traffics that need to go through each communication link is same. However, this is not likely to be true and it is possible that some communication link carries more traffic than others.



(a) network (b) assigned colors

Figure 6: Simple example: unweighted coloring is inefficient.

Consider a simple example of multihop wireless network composed of k source and destination pairs (s_i, t_i) as shown in Figure 6(a). For simplicity of our presentation, we assume that every node in the network can transmit at a bps if it uses all time slots. Observe that we need at least $k+1$ colors, which can be obtained by assigning color i to links $s_i v_1$ and $v_2 t_i$, and color $k+1$ to link $v_1 v_2$ as in Figure 6(b). This implies that communication link $v_1 v_2$ can transmit once every $k+1$ time slots. However, the path between each source destination pair needs to go through link $v_1 v_2$. Thus, link $v_1 v_2$ becomes the bottleneck and the overall network throughput is only $\frac{a}{k+1}$ bps. For each source destination pair, its throughput is approximately $\frac{a}{k \cdot (k+1)}$ bps, which is inefficient. Thus, we need to generalize the coloring that can take the traffic rate on each communication link into account. In this paper, we use the *weighted coloring* to capture this, which is defined as follows.

DEFINITION 1. *Given a graph $G = (V, E)$ where V is the set of vertices and E is the set of links. Every link $e_i \in E$ has an integral weight $w_i \geq 0$. A weighted link coloring is an assignment of at least w_i distinct colors to each link e_i such that no two links sharing the same color interfere with each other.*

By introducing the notation of weighted coloring, we can assign different weight to different communication links. For example, given a set of k flow requirements f_i from s_i to t_i, $1 \leq i \leq k$, a certain routing algorithm will determine the routing path for each flow. The weight of a link e is then the total flow passing through e divided by the bandwidth $c(e)$ of link e, *i.e.*, $w_e = \frac{\sum_{f_i : f_i \ using \ e} f_i}{c(e)}$. Let us see how the weighted coloring can help to improve the throughput using the example shown in Figure 6. By assigning weight 1 to each link $s_i v_1$, $v_2 t_i$ for $1 \leq i \leq k$ and k to $v_1 v_2$, obviously a valid $2k$ coloring can be obtained. It is not difficult to observe that the total throughput is now $a/2$ bps and each communication pair has a throughput of $a/2k$. This increases the throughput obtained from the unweighted coloring by an order of k. Following, we show how to obtain a valid weighted coloring based on the unweighted coloring.

Algorithm 6 Weighted Coloring Algorithm Based on Unweighted Coloring Algorithm $\mathcal{A}$

Input: A communication graph $G = (V, E)$ with weight on each link and an unweighted coloring algorithm $\mathcal{A}$.
Output: A valid coloring of the links.

1: Build the conflict graph F_G based on original graph G and interference model. Assign weight $w_{i,j}$ to vertex $l_{i,j} \in F_G$.
2: Construct a new conflict graph F'_G from F_G as follows: for each vertex $l_{i,j}$ with weight $w_{i,j}$, we create $w_{i,j}$ vertices, $l_{i,j}^1$, $l_{i,j}^2$, ..., $l_{i,j}^{w_{i,j}}$ and add them to F'_G. Add to graph F'_G the edges connecting $l_{i,j}^a$, $l_{i,j}^b$ for $1 \leq a < b \leq w_{i,j}$. Add to graph F'_G an edge between $l_{i,j}^a$ and $l_{p,q}^b$ if and only if there is an edge between $l_{i,j}$ and $l_{p,q}$ in graph F_G.
3: Run the unweighted vertex coloring algorithm $\mathcal{A}$ on F'_G.
4: Assign link $l_{i,j}$ all the colors that are used by $l_{i,j}^k$ for $1 \leq k \leq w_{i,j}$ in F'_G.

We then show that Algorithm 6 has a performance guarantee that is not worse than that of the unweighted coloring algorithm $\mathcal{A}$.

THEOREM 15. *If $\mathcal{A}$ uses at most α times of the optimal colors for unweighted coloring, then Algorithm 6 also needs at most α times of the optimal colors for weighted coloring.*

PROOF. Notice that for any valid weighted coloring for F_G, $l_{i,j}$ is assigned at least $w_{i,j}$ colors. By assigning each vertex $l_{i,j}^k$ in F_G' a distinct color that is assigned to $l_{i,j}$, we obtain a valid unweighted coloring for F_G'. Thus, $\chi(F_G') \leq \chi(F_G)$. Here $\chi(F_G')$ is the minimum number of colors needed for unweighted coloring in F_G' and $\chi(F_G)$ is the minimum number colors needed for weighted coloring in F_G. Since $\mathcal{A}$ will return a coloring with at most $\alpha \cdot \chi(F_G')$ colors, Algorithm 6 produces a coloring with at most $\alpha \cdot \chi(F_G') \leq \alpha \cdot \chi(F_G)$ colors. This finishes the proof. □

The basic idea of Algorithm 6 is to create a clique of size $w_{i,j}$ for each link $l_{i,j}$ and color the new graph using unweighted coloring method $\mathcal{A}$. Although this gives a general framework to design weighted coloring, its time-complexity could be large if the weight is large. Fortunately, Algorithm 6 could be simplified without much overhead compared to the unweighted algorithm: the main idea is to assign colors for one link at once: instead of assigning one time-slot to a link l_k, we assign w_k time-slots to link l_k when process link l_k. As an example, we modify the Algorithm 4 to obtain a fast weighted coloring (Algorithm 7). Following we show that Algorithm 7 has the same performance guarantee as Algorithm 4.

Algorithm 7 Fast Distributed Weighted Coloring Algorithm

Input: A communication graph $G = (V, E)$.
Output: A valid coloring of links in the communication graph.

1: Node v_i computes a subset, say H_i, of all communication links containing v_i such that link $l_{i,j} \in H_i$ if and only if $r_i > r_j$.
2: **while** node v_i failed to obtain the channel **do**
3: Node v_i monitors the channel and competes for the channel.
4: **for** each link $l_{i,j} \in H_i$ **do**
5: Color link $l_{i,j}$ with the first fit $w_{i,j}$ colors that are not used by any link that interferes or is interfered by $l_{i,j}$. Here, the assigned colors are not required to be continuous.
6: Broadcasts the message Color(i, j, k) to each head of links that conflict with $l_{i,j}$.

THEOREM 16. *Algorithm 7 finds a coloring that needs at most $O(\log(\psi) + 1)$ times of optimum.*

PROOF. Let $\mathcal{A}_w$ be the coloring algorithm by applying Algorithm 6 based on Algorithm 4. Observe that the coloring of $\mathcal{A}_w$ is nondeterministic, *i.e.*, the output could be different because of the randomization introduced by the different processing time of different nodes. However, it is true that the output of Algorithm 7 is one of the possible outputs of $\mathcal{A}_w$. From Theorem 15, any coloring output by $\mathcal{A}_w$ is at most $O(\log(\psi) + 1)$ times the optimal. Thus, Algorithm 7 computes a coloring that needs at most $O(\log(\psi)+1)$ times optimal color. □

Similarly, we can modify Algorithm 1 and Algorithm 3 to obtain efficient weighted coloring methods with the same time complexities and approximation ratios. The details are omitted here.

5.2 Necessary and Sufficient Conditions for Schedulable Flows

Similar to [1,17,21], we also make the connection with flows on the links of a wireless network G and the link scheduling. We will give both a necessary and a sufficient condition on the link flows such that an interference-free link scheduling is feasible. Recall that we use $\ell(e)$, $c(e)$ to denote the load and the capacity of a link e respectively. From Lemma 3 and Theorem 4, it follows that

THEOREM 17. *Under the RTS/CTS model, any link flow ℓ that permits an interference-free link scheduling must satisfy the constraint $\frac{\ell(e)}{c(e)} + \sum_{e' \in I^{\geq}(e)} \frac{\ell(e')}{c(e')} \leq 2C_1$. On the other hand, if $\frac{\ell(e)}{c(e)} + \sum_{e' \in I^{\geq}(e)} \frac{\ell(e')}{c(e')} \leq 1$, then any link flow ℓ permits an interference-free link scheduling.*

Similarly, under the fPrIM Model, we have

THEOREM 18. *Under the fPrIM model, any link flow ℓ that permits an interference-free link scheduling must satisfy the following constraint $\frac{\ell(e)}{c(e)} + \sum_{e' \in I^{in}(e)} \frac{\ell(e')}{c(e')} \leq \lceil \frac{2\pi}{\arcsin \frac{\gamma-1}{2\gamma}} \rceil$. On the other hand, if $\frac{\ell(e)}{c(e)} + \sum_{e' \in I^{in}(e)} \frac{\ell(e')}{c(e')} \leq 1$, then any link flow ℓ permits an interference-free link scheduling.*

The proofs of the above theorems are similar to those of [1, 17, 21] for other interference and networking models, and are thus omitted here due to space limit. Similar theorems can be obtained for networks with multiple channels and multiple radios.

6. PERFORMANCE EVALUATION

We evaluate the performances of our new link scheduling algorithms for RTS/CTS model by conducting simulations with random networks.

Network Settings: In these simulations, we randomly generate n wireless nodes uniformly in a 10×10 unit region. The transmission range is randomly drawn from 1.8 to 2 unit, while the interference range is randomly set to be 1.5 to 2 times of its transmission range. Typically, a unit represents about 50 meters here. We assume there is a sink (or an access point) in the network, all traffics are towards it. The sink is placed in the center of the region in the simulations. We vary the node number n from 40 to 200. For each number n, 100 vertex sets (networks) are randomly generated. Given a sampled network, we not only test the number of colors and the network throughput resulted by our various link scheduling algorithms, but also count the number of messages and rounds used by the distributed algorithms. The average of these performances over all these 100 randomly sampled networks are reported. For each source, we run the classical shortest path algorithm to determine the traffic route. Notice that our scheduling algorithms do not rely on any particular routing algorithms, here the shortest path routing is used as an example.

In the first scenario, we assume the system does not know the volume of each traffic. So it is an unweighted case where we need to assign one color for each link involved in the traffics. We test our centralized and two distributed algorithms (Algorithm 1 [Cent], Algorithm 3 [Dist-1], and Algorithm 4 [Dist-2]). The simulation results are reported in Figure 7. First, for the number of colors and the throughput, three algorithms have similar performances. When the node number increases, more colors are needed and the throughput decreases. The centralized algorithm has the best throughput while the fast distributed algorithm has the worst, as our expectation. For both distributed algorithms we also count the number of messages and rounds used. It shows that Dist-1 algorithm used much more messages and rounds than Dist-2 (fast distributed algorithm). The large number of rounds and messages needed by Dist-1 is due to the first two steps in Algorithm 3, which collect all communication links in H_i and M_i. The large number of rounds of Dist-1 is mainly due to conflicts among messages for collecting information. Notice that two adjacent links in the conflict graph need to compete for the channel first. After a node v_i obtained the channel, it uses a unit of time to assign colors to all links in H_i and inform other interfering links about the coloring used.

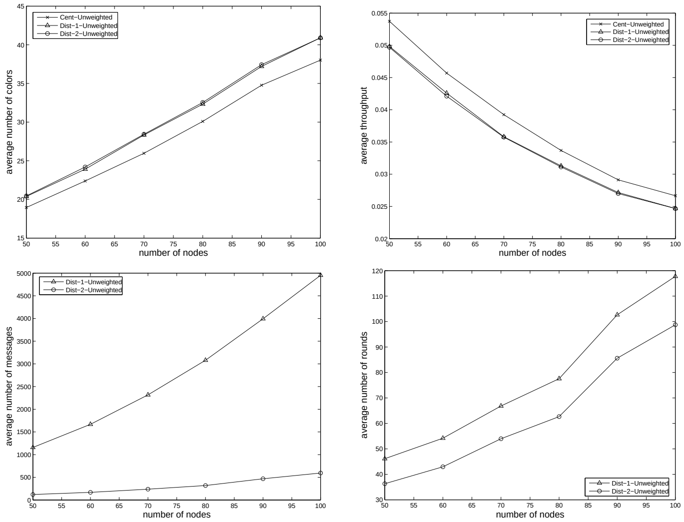


Figure 7: Scheduling without traffic load information.

In the second scenario, we randomly draw the traffic produced by each node from 1 to 10 units. Then for each link $l_{i,j}$, its weight $w_{i,j}$ is the total volumes of traffics that need to go through it, which could be 0. The simulation results are given in Figure 8. The throughput of weighted methods are much better than those of unweighted methods. Our centralized and distributed methods have similar throughput.

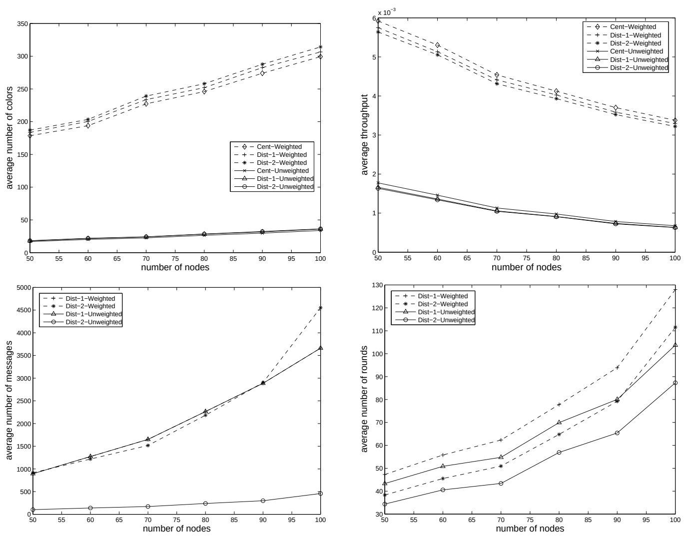


Figure 8: Scheduling with nonunform traffic load.

7. RELATED WORK

Scheduling has been studied extensively in the past few years due to its application for assigning time slots in TDMA MAC protocols that eliminate collision, guarantee fairness. Scheduling can be reduced to different coloring problems: *edge coloring* and *vertex coloring*.

Edge coloring, in which every edge corresponds to a valid communication link, is a natural way to capture the link scheduling problem. An edge coloring is *valid* if no two incident edges share the same color. Vizing's theorem [4] states that a valid edge coloring for an *indirected* graph can be obtained by using at most $\Delta+1$ colors, where Δ is the maximum node degree in the graph. On the other hand, any edge coloring needs at least Δ colors. Any edge coloring that uses $\Theta(\Delta)$ colors is close to the optimal. Panconesi and Srinivasan [25] proposed a randomized distributed edge coloring method that uses at most $2\Delta+1$ colors.

To some extent, this captures some transmission restrictions in wireless ad hoc and sensor network in which no node can receive or send at the same time slot, but it did not address some other interferences such as secondary interference. When one has a valid edge coloring, it can be easily mapped to a TDMA scheduling. However, it is possible that two communication links sharing the same color still interfere with each other in a wireless network. In order to remedy this, Gandham *et al.* [10] proposed to use a two phase scheduling method: in the first phase, a distributed valid edge coloring is obtained; in the second phase, a valid scheduling taken into account the secondary interference is obtained. In essence, [10] is based on the protocol interference model. The overall scheduling in [10] only provided a performance guarantee when the conflicting links form a tree. In [15], Jain *et al.* proposed a new concept *conflict graph* that captures the interference in a wireless networks.

Vertex coloring is one of the most fundamental NP-hard problems in graph theory and has been thoroughly studied. A vertex coloring is *valid* iff any two adjacent vertices receive different colors. The minimum number that is needed for a valid vertex coloring for a graph G is known as the *chromatic number* $\chi(G)$. It is known that for general graph, the chromatic number cannot be approximated within $n^{1-\varepsilon}$ for any $\varepsilon > 0$, unless ZPP=NP [9]. For vertex coloring of a general graph G, it was proved that, every graph G can be colored using $\delta(G)+1$ colors. Then Hochbaum [14] presented a method to find the value of $\delta(G)$ and color G using $\delta(G)+1$ colors in $O(|V|+|E|)$ time. Ramanathan [26] proposed a unified framework for TDMA, FDMA and CDMA based multi-hop wireless networks. They also proposed a timeslot assignment to edges; the number of timeslots required is at most $O(\theta)$ times the optimum, where θ is the thickness of a graph, *i.e.*, the minimum number of planar graphs into which the network can be decomposed. Krumke *et al.* [19] proposed efficient approximation algorithms for the distance-2 vertex coloring problem for various geometric graphs including (r,s)-civilized graphs, planar graphs, graphs with bounded genus, etc. In [20], Kumar *et al.* studied packet-scheduling under RTS/CTS interference model and gave polylogarithmic/constant factor approximation algorithms for various families of disk graphs and randomized near-optimal approximation algorithms for general graphs.

Several distributed algorithms that use $O(\Delta)$ colors have been proposed in literatures. A $(\Delta+1)$-coloring can be computed in time $O(\log n+\Delta)$ [24] or $O(\Delta\log n)$ [11]. In [22], Maraco *et al.* proposed a distributed algorithm that computed an $O(\Delta)$-coloring in time $O(\log n)$. All of the above distributed algorithms do not take the interference into account and is based on the message passing model, which implies that the actual time used in a wireless environment could be much larger [23]. Recently, Moscibroda *et al.* [23] proposed an $O(\Delta)$ distributed coloring method with time-complexity $O(\Delta\log n)$. It is worth to point out that the coloring in [23] considered a simple interference model and the time is close to time needed in practice. However, the coloring in [23] is based on the assumption that the wireless ad hoc network can be modeled as a unit disk graph (UDG), *i.e.*, their method will return a coloring that only guarantees that any nodes that are adjacent in the UDG will get different colors; nodes that are not adjacent in UDG may get the same color. In addition, they assumed that all nodes have the same transmission range and same interference range as its transmission range. This is different from the interference-free scheduling studied in this paper.

Kodialam and Nandagopal [16] studied the effect of interference on the achievable rate region in multi-hop wireless networks. They treated the interference models as linear constraints and solve the flow problem using linear program. In [17], the same authors considered the problem of jointly routing the flows and scheduling transmissions to achieve a given rate vector using the protocol model of interference. They developed necessary and sufficient conditions for the achievable rate vector. They formulated the problem as a linear programming problem and implemented primal-dual algorithms for solving the problem. The scheduling problem is solved as a graph edge-coloring problem using existing greedy algorithms. In [18], they extended their work to the multi-radio multi-channel wireless mesh networks.

Kumar *et al.* [21] developed analytical performance evaluation models and distributed algorithms for routing and scheduling which incorporate fairness, energy and dilation (path-length) requirements and provide a unified framework for utilizing the network close to its maximum throughput capacity. Alicherry *et al.* [1] mathematically formulated the joint channel assignment and routing problem in multi-radio mesh networks, and established necessary and sufficient conditions under which interference free link communication schedule can be obtained and designed an simple greedy algorithm to compute such a schedule. Notice that the studied network in [1] is restricted to be a UDG, *i.e.*, the uniform interference range is assumed to be a fixed multiple of the uniform communication range.

Recently, Chen *et al.* [5, 6] also studied the cross-layer optimization of congestion control and routing together with scheduling problem under both primary and secondary interference.

8. CONCLUSION

In this paper, we considered the problem of obtaining a good interference-aware link scheduling for a wireless network to maximize the throughput of the network. We used the link coloring to resolve this and assumed a general model for wireless networks, *i.e.*, nodes could have different transmission ranges and different interference ranges, and a link uv may not exist even if $\|uv\|$ is less than the transmission range of node u. We presented both centralized algorithms and efficient distributed algorithms that use timeslots within a constant factor of the optimum. We also pointed out that the simple link coloring does not imply a good throughput, and then proposed a general weighted link coloring problem and gave efficient algorithms to obtain TDMA link scheduling with proven performances. We also conducted extensive simulations to study the performances of our algorithms. Our theoretical results are corroborated by our simulation studies.

There are still a number of challenging questions left for future research. The first question is how to efficiently collect the information about the interfering links of a given link in a wireless networking environment. This is not an issue in the previous studies since they assumed a unit disk graph model and assumed the same interference range for all nodes. The second question is how to improve the overall time complexity of our distributed algorithms. The results presented in [23] may give some insights on this but it is not obvious because of the model used in this paper is more complicated than the model used in [23]. We suspect the existence of polylogarithmic time distributed algorithms for problems studied in this paper under the unstructured environment [23]. The third question is to study the link scheduling in an asynchronized environment. We believe that our methods still apply with small modifications. The last but not the least question is to study the link scheduling in a dynamic environment where the traffic load on links could have some small changes.

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