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Energy-Efficient Localized Routing in Random Multihop Wireless Networks

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***Abstract*— A number of energy-aware routing protocols were proposed to seek the energy efficiency of routes in multihop wireless networks. Among them, several geographical localized routing protocols were proposed to help making smarter routing decision using only local information and reduce the routing overhead. However, all proposed localized routing methods cannot guarantee the energy efficiency of their routes. In this paper, we first give a simple localized routing algorithm, called *Localized Energy-Aware Restricted Neighborhood routing* (LEARN), which can guarantee the energy efficiency of its route if it can find the route successfully. We then theoretically study its critical transmission radius in random networks which can guarantee that LEARN routing finds a route for any source and destination pairs asymptotically almost surely. We also extend the proposed routing into three-dimensional (3D) networks and derive its critical transmission radius in 3D random networks. Simulation results confirm our theoretical analysis of LEARN routing and demonstrate its energy efficiency in large scale random networks.**

I. Introduction

Energy conservation and *scalability* are probably two most critical issues in designing protocols for multihop wireless networks, because wireless devices are usually powered by batteries only and have limited computing capability while the number of such devices could be large. In this paper we focus on designing routing protocols for multihop wireless networks which can achieve both energy efficiency by carefully selecting the forwarding neighbors and high scalability by using only local information to make routing decisions.

Numerous energy aware routing protocols [1]–[8] have been proposed recently using various techniques (transmission power adjustment, adaptive sleeping, topology control, multipath routing, directional antennas, etc). Most of the proposed energy-aware routing methods take into account the energy-related metrics instead of traditional routing metrics such as delay or hop count. To select the optimal energy route, those methods usually need the global information of the whole network, and each node needs to maintain a routing table as protocol states. In opposition to these table-driven routing protocols, several stateless routing protocols, particularly, localized geographic routing protocols [9]–[11] have been proposed to improve the scalability. In those localized routing protocols, with the assumption of known position information, the routing decision is made at each node by using only local neighborhood information. They do not need dissemination of route discovery information, and no routing tables are maintained at each node. Previous localized routing protocols are not energy efficient, *i.e.*, the total energy consumed by their route could be very large compared with the optimal. Recently, several energy-aware localized routing protocols [5], [7], [8], [12], [13] take the energy consumption into consideration during making the routing decisions. However, all of them cannot theoretically guarantee energy efficiency of their routes in the worst case. Actually, Kuhn *et al.* [14] proved by constructing an example where the path found by *any* deterministic localized routing protocol to connect two nodes s and t has energy consumption asymptotically at least $\Theta(OPT^2)$ in the worst case. Here OPT is the energy consumption of the least energy cost path connecting s and t.

In this paper, we study an energy-efficient localized routing protocol and its critical transmission radius for random wireless networks. Our main contributions are as follows. (1) We propose a new localized routing protocol, called *localized energy-aware restricted neighborhood routing* (LEARN) in Section III. In LEARN, whenever possible, the node selects the neighbor inside a restricted neighborhood (defined by a parameter α) that has the largest energy mileage (i.e., the distance traveled per unit energy consumed) as the next hop node. (2) We theoretically prove that LEARN is energy efficient, i.e., when LEARN routing finds a path from the source node to the target node, the total energy consumption of the found path is within a constant factor of the optimum, in Section IV. LEARN routing is the *first* localized routing which can *theoretically* guarantee the energy efficiency of its routes. (3) We theoretically prove (in Section V) that for a random network, formed by nodes that are produced by a Poisson distribution with rate n over a compact and convex region Ω with unit area, when the transmission radius $r = \sqrt{\frac{\beta \ln n}{\pi n}}$ for some $\beta > \frac{\pi}{\alpha}$, our LEARN routing protocol will find a route for any pair of nodes asymptotically almost surely (*a.a.s.*). When the transmission radius $r = \sqrt{\frac{\beta \ln n}{\pi n}}$ for some $\beta < \frac{\pi}{\alpha}$, the LEARN routing protocol will *not* be able to find a route for any pair of nodes asymptotically almost surely. (4) We extend our proposed routing method to 3D routing protocol and derive the critical transmission radius of 3D LEARN routing in Section VI. (5) We conducted extensive simulations to study the critical transmission radius and routing performance of LEARN in random networks. The simulation results, provided as *Supplemental Material*, confirm our theoretical analysis.

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II. Network Model and Routing Problem

A. Network Model and Energy Mileage

Network Model: We consider a set V of n wireless devices (called nodes hereafter) distributed in a compact and convex region Ω. Typically, the region Ω is a unit-area square or a unit-area disk. We assume that each node knows its position information either through a low-power GPS receiver or some other ways (such as localization algorithms or location services). By a single-hop broadcasting, each node i can gather the location information of all nodes within its transmission radius. We assume that every node has a uniform transmission radius r[1]. The multihop wireless network is then modeled by a communication graph $G(V,r)$, where two nodes u and v are connected in $G(V,r)$ if and only if their Euclidean distance is at most r. Hereafter, we use $\|u-v\|$ to denote the Euclidean distance between node u and node v. For an undirected link $uv \in G(V,r)$, we use $\|uv\|$ to denote its length. We also assume that each node i can dynamically adjust its transmission power based on the neighboring node it wants to communicate.

Energy Model: We further assume that the energy needed to support the transmission of a unit amount of data over a link uv is $\mathbf{c}(\|uv\|)$, where $\mathbf{c}(x)$ is a non-decreasing function on x. We then define a concept of energy mileage.

Definition 1: Given a energy model $\mathbf{c}(x)$, *energy mileage* is the ratio between the transmission distance and the energy consumption of such transmission, *i.e.*, $\frac{x}{\mathbf{c}(x)}$.
Let $\mathbf{r}_0$ be the value such that $\frac{\mathbf{r}_0}{\mathbf{c}(\mathbf{r}_0)} = \max_x \frac{x}{\mathbf{c}(x)}$. We call $\frac{\mathbf{r}_0}{\mathbf{c}(\mathbf{r}_0)}$ as the *maximum energy mileage*[2] under energy consumption model $\mathbf{c}(x)$. We assume that the energy mileage $\frac{x}{\mathbf{c}(x)}$ is an increasing function when $x < \mathbf{r}_0$ and is a decreasing function when $x > \mathbf{r}_0$. For example, in the literature it is commonly assumed that $\mathbf{c}(\|uv\|) = E_0(\|uv\|^{\gamma} + c)$, where E_0, γ, and c are constants depending on the transmission environment and device. For the sake of the analysis, it is often assumed that $E_0 = 1$ and $\gamma = 2$, i.e., $\mathbf{c}(\|uv\|) = \|uv\|^2 + c$. Under this specific energy consumption model, the maximum energy mileage is $\mathbf{r}_0 = \sqrt{c}$. Notice that when $\mathbf{r}_0 > r$, the best length of a forwarding link is r. Our proposed localized routing will greedily select the neighbor who can maximize the energy mileage as the forwarding node.

Notice that the concept of energy mileage is not completely new. A similar concept called *power progress* was first proposed by Kuruvila *et al.* in [8], which is the ratio between the energy consumption and the distance progress towards the destination. Their localized routing method greedily picks the neighbor, who minimizes the power progress, as the reply node. Later, this concept was generalized to *cost to the progress ratio* in [16] and used in some localized routing methods [12], [13]. Thus, the major contribution of this article is not another energy-aware localized routing method based on energy mileage concept but the theoretical analysis of energy efficient guarantee and delivery guarantee of the new routing method which applies energy mileage in a restricted region during the relay selection.

[1]In this paper, we consider the transmission range of each node a perfect disk (defined by radius r), which may not be true in reality. However, such simplification allows us to perform the proposed theoretical analysis and provide predictable performance of the proposed protocol. If the transmission range is not a disk, we can assume that r is the radius of the largest disk inside the range.

[2]Here we assume that the derivative of function $d(\frac{\mathbf{c}(x)}{x})/dx = \frac{\mathbf{c}'(x)x - \mathbf{c}(x)}{x^2}$ is monotone increasing, thus, $\mathbf{r}_0$ is unique.

B. Problem Specification: Localized vs Energy Efficient

In this paper, we focus on designing a new localized and energy efficient routing method. Here we give the formal definitions of *localized routing* and *energy efficient routing*.

Definition 2: A routing protocol $\mathcal{A}$ is said to be a *localized* protocol if, given the information of the source node s and the target node t and the k-hop neighborhood information, the current node u can decide which neighboring node v to forward the message. Here k is a constant, usually 1 or 2.

Given a routing method $\mathcal{A}$, let $\mathbf{P}_{\mathcal{A}}(s,t)$ be the path found by $\mathcal{A}$ to connect the source node s and the target node t. Assume that path $\mathbf{P}_{\mathcal{A}}(s,t) = v_0 v_1 v_2 \cdots v_{k-1} v_k$ where $v_0 = s$ and $v_k = t$. Then the total energy consumption of this path is $\sum_{i=1}^{k} \mathbf{c}(\|v_{i-1}v_i\|)$.

Definition 3: A routing method $\mathcal{A}$ is called *energy efficient* if for every pair of nodes s and t, the energy consumption of path $\mathbf{P}_{\mathcal{A}}(s,t)$ is within a constant factor of the least energy-consumption path connecting s and t in the network.

For a general network, it was shown in [14] that no *deterministic* localized routing method is energy efficient. By constructing a simple network example (Figure 8 in [14]), Kuhn *et al.* proved that any deterministic geometric routing algorithm could have cost (either hop-count, Euclidean distance, or energy-consumption) $\Omega(OPT^2)$ in the worst case, where OPT is the optimum cost. Therefore, we will concentrate on designing a localized routing method that is energy efficient with high probability for random networks. Here a routing method is energy efficient with high probability if (1) with high probability, the routing method will find a path successfully; and (2) with high probability, the found path is energy efficient.

III. Energy Efficient Localized Routing

In this section, we describe in detail our energy-efficient localized routing method, called *localized energy-aware restricted neighborhood routing* (LEARN), which is a variation of classical greedy routing. In greedy routing, current node u selects its next hop neighbor based purely on its distance to the destination, *i.e.*, it sends the packet to its neighbor who is closest to the destination. However, such choice might not be the most energy-efficient link locally, and the overall route might not be globally energy-efficient too. The definition of energy mileage provides us the insight in designing energy-efficient routing. Whenever possible, the forwarding link that has larger energy mileage should be used. In addition, to save the energy consumption, the total distance traveled should be as small as possible. Thus, we introduce a restricted region to restricting the forwarding direction. Intuitively, our routing protocol will work as follows.

- The current intermediate node u with a message first finds the "best" neighbor v among all neighbors w inside a restricted area (*i.e.,* angle $\angle wut \leq \alpha$ for a parameter

$\alpha < \pi/3$) as shown in Figure 1(a). Here we define the "best" neighbor as the node v such that its energy mileage $\frac{\|uv\|}{\mathbf{c}(\|uv\|)}$ is maximum among all such neighbors (*i.e.*, $\|uv\|$ is the nearest to $\mathbf{r}_0$ and $\eta_1 \mathbf{r}_0 \leq \|uv\| \leq \eta_2 \mathbf{r}_0$ where η_1 and η_2 are two constant parameters). Recall that $\mathbf{r}_0$ is the best link length that achieves the maximum energy mileage.

- If there is no neighbor inside the restricted area (or $\mathbf{r}_0 \geq r$), current node u finds the node v inside the 2α-sector region (as shown in Figure 1(b)) with the minimum $\|t - v\|$. The use of the angle α (restricting the forwarding direction) in our algorithm is to bound the total distance of the routing path. This can help us to prove the energy-efficiency of the route.
- When there is no neighbor in the 2α-sector region, classical greedy routing (as shown in Figure 1(c)) or face routing [9], [10] can be applied.

Notice that the ideas of using restricted region and energy mileage are not completely new. Restricted region with an angle has been used in some localized routing methods, such as nearest/farthest neighbor routing [17], while concepts similar to energy mileage have been used in some energy-aware localized routing methods [8], [12], [13]. However, combining both of these techniques to guarantee energy efficiency of paths has never been done before.

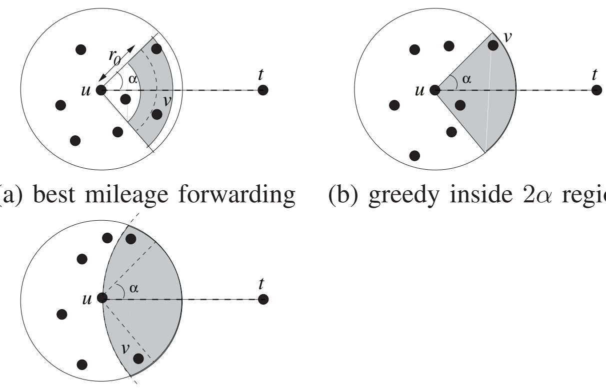
Algorithm 1 illustrates our localized energy-aware routing protocol. In our protocol, there are four input parameters: (1) $\alpha < \pi/3$ is an adjustable parameter to define the 2α-sector restricted forwarding region; (2) η_1 and η_2 are two constant parameters to control the restricted forwarding region around $\mathbf{r}_0$ if $\mathbf{r}_0 < r$, usually $\eta_1 < 1$ and $\eta_2 > 1$; and (3) $\mathbf{r}_0$ is the link length with maximum energy mileage which can be derived from energy model $\mathbf{c}(x)$ (e.g., $\mathbf{r}_0 = \sqrt{c}$ for energy model $\mathbf{c}(x) = x^2 + c$). For example, the following setting of these parameters can be used for energy model $\mathbf{c}(x) = x^2 + c$: $\alpha = \frac{\pi}{4}$, $\mathbf{r}_0 = \sqrt{c}$, $\eta_1 = 1/2$ and $\eta_2 = 2$. To make the later analysis easier, we call the routing algorithm LEARN if *no* Greedy routing and *no* Face routing is used when no node v satisfying that $\angle vut \leq \alpha$. If greedy routing is applied afterward, then the routing protocol is called LEARN-G. Furthermore, if the Face routing is used at the end to get out of the local minimum, the routing protocol is called LEARN-GF.

To the best of our knowledge, LEARN is the first localized routing protocol that has theoretical guarantee of energy efficiency of its routes with high probability. Next, we first show that the found path by LEARN is energy efficient with high probability (Section IV), then prove that LEARN can find a path successfully with high probability if the transmission radius is larger than certain value (Section V).

IV. Path Length and Energy Efficiency of LEARN

We first prove that the total length of the path found by LEARN routing protocol is within a constant of the optimum if LEARN successfully finds a path.

Theorem 1: When LEARN routing indeed finds a path $\mathbf{P}_{LEARN}(s,t)$ from the source s to the target t, the total Euclidean length of the found path is at most $\delta\|t - s\|$ where $\delta = \frac{1}{1-2\sin\frac{\alpha}{2}}$, thus, a constant factor of the optimum.



(a) best mileage forwarding (b) greedy inside 2α region (c) greedy forwarding

Fig. 1. Illustrations of LEARN routing: (a) energy efficient forwarding in a restricted forwarding region, (b) greedy forwarding in the 2α-sector region, (c) classic greedy forwarding when the sector region is empty.

Algorithm 1 LEARN: Localized Energy-Aware Restricted Neighborhood Routing

Input: Three parameters $0 < \alpha < \frac{\pi}{3}$ and $\eta_1 < 1 < \eta_2$ difining the restricted region, and the best energy milelage distance $\mathbf{r}_0$.

while node u receives a packet with destination t **do**
  if $\|t - u\| \leq r$, *i.e.* t is a neighbor of u **then**
    Node u forwards the data to t directly and return.
  else if ($\mathbf{r}_0 < r$) and ($\exists v$ with $\eta_1\mathbf{r}_0 \leq \|uv\| \leq \eta_2\mathbf{r}_0$ and $\angle vut \leq \alpha$) **then**
    Node u forwards the packet to such a neighbor v such that $|\|uv\| - \mathbf{r}_0|$ is minimized. See Figure 1(a).
  else if $\exists v$ with $\|t - v\| < \|t - u\|$ and $\angle vut \leq \alpha$ **then**
    Node u forwards the packet to the node v with the minimum $\|t - v\|$. See Figure 1(b).
  else if $\exists v$ with $\|t - v\| < \|t - u\|$ **then**
    Node u forwards the packet to the node v with the minimum $\|t - v\|$. In other words, node u applies the traditional Greedy routing. See Figure 1(c).
  else
    Node u simply drops the packet, or applies the Face routing method to guarantee the delivery.
  end if
end while

Proof: We prove it by induction on the number of hops of the found path. It is clearly true when the path has only one-hop. Assume that it is true for the path with $(k-1)$-hops. Then consider any path $v_0 v_1 v_2 \cdots v_{k-1} v_k$ with k-hops. By induction, the length of path $v_1 v_2 \cdots v_{k-1} v_k$ is at most $\delta\|v_k - v_1\|$. Then it is sufficient to show that $\|v_0 v_1\| + \delta\|v_k - v_1\| \leq \delta\|v_k - v_0\|$.

As shown in Figure 2, let $\angle v_0 v_k v_1 = 2\sigma$ and $\angle v_1 v_0 v_k = \theta \leq \alpha < \pi/3$. Then a simple geometry computation shows that $\frac{\|v_0 v_1\|}{\|v_k - v_0\| - \|v_k - v_1\|} = \frac{\|v_0 v_1\|}{\|v_0 - u\|} = \frac{\sin\phi}{\sin\chi} = \frac{\sin(\sigma+\pi/2)}{\sin(\frac{\pi}{2}-\sigma-\theta)} = \frac{\cos\sigma}{\cos(\sigma+\theta)}$. Notice that by our routing protocol, we have $v_0 v_k$ is the longest link in triangle $v_0 v_1 v_k$. Thus, it is easy to show that we need $\sigma < \frac{\pi}{2} - \theta$ and $\sigma < \frac{\pi-\theta}{4}$. Simple computation shows that $\frac{\cos\sigma}{\cos(\sigma+\theta)} \leq \frac{1}{1-2\sin\frac{\theta}{2}} \leq \frac{1}{1-2\sin\frac{\alpha}{2}}$ for $\sigma < \min(\frac{\pi}{2} - \theta, \frac{\pi-\theta}{4})$. Thus, $\|v_0 v_1\| + \delta\|v_k - v_1\| \leq$

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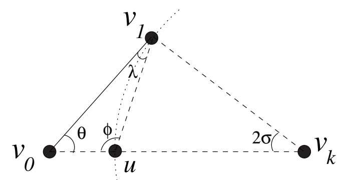


Fig. 2. Illustration for the proof of Theorem 1.

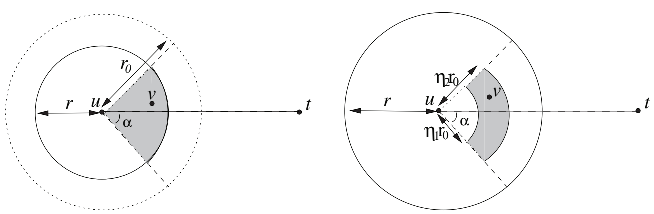
$\delta\|v_k - v_0\|$. This completes the proof. ■

We then show that our LEARN routing protocol is indeed energy efficient when it finds the path successfully. We prove it by two separate cases: $\mathbf{r}_0 \geq r$ or $\mathbf{r}_0 < r$ and use the following lemma whose proof is given in *Supplemental Material*.

Lemma 2: We have $\angle wut \leq \alpha$ if $\|uv\| + \|vw\| \leq r$, $\angle vut \leq \alpha$ and $\angle wvt \leq \alpha$.

Theorem 3: When LEARN routing indeed finds a path $\mathbf{P}_{LEARN}(s,t)$ from the source s to the target t and $\mathbf{r}_0 \geq r$, the total energy consumption of the found path is within a constant factor of the optimum.

Proof: Recall that when $\mathbf{r}_0 \geq r$, for any intermediate node u with packets to forward to destination t, our LEARN routing protocol will select the neighbor v with $\angle vut \leq \alpha$ and $\|t - v\|$ is minimized. See Figure 3(a) for illustration.



(a) Case 1: $\mathbf{r}_0 \geq r$ (b) Case 2: $\mathbf{r}_0 < r$

Fig. 3. Illustrations of the proof of energy efficiency: two cases when LEARN routing selects a node inside the restricted region.

Assume that the path $v_0 v_1 v_2 \cdots v_{k-1} v_k$ is found by our routing protocol to connect source node $s = v_0$ and destination node $t = v_k$. We first prove that for every two consecutive links $v_{i-1}v_i$ and $v_i v_{i+1}$, $\|v_{i-1}v_i\| + \|v_i v_{i+1}\| > r$. Assume that this is not true, *i.e.*, there are 3 consecutive nodes, say u, v, and w on the path with $\|uv\| + \|vw\| \leq r$. Obviously, w is neighbor of u since $\|uw\| \leq \|uv\| + \|vw\| \leq r$. From Lemma 2, we know that $\angle wut \leq \alpha$. Additionally, by the routing protocol we know that $\|t - w\| < \|t - v\|$ and $\|t - v\| < \|t - u\|$. These contradict the selection of node v by node u for forwarding: it should select the node that is closest to destination t and clearly node v is not (since w is closer to t than v).

Notice that since $\mathbf{r}_0 \geq r$, then for any path connecting v_0 and v_k, its energy consumption is at least $\frac{\|v_0 - v_k\|}{r}\mathbf{c}(r)$. This can be proved as follows. Let x_i, $1 \leq i \leq k$ be the ith link length of the path. Then its total energy consumption is $\sum_{i=1}^{k} \mathbf{c}(x_i) = \sum_{i=1}^{k} \frac{\mathbf{c}(x_i)}{x_i} \cdot x_i \geq \sum_{i=1}^{k} \frac{\mathbf{c}(r)}{r} \cdot x_i = \frac{\mathbf{c}(r)}{r} \sum_{i=1}^{k} x_i \geq \frac{\|v_0 - v_k\|}{r}\mathbf{c}(r)$.

Let $x_i = \|v_{i-1}v_i\|$. Theorem 1 implies that $\sum_{i=1}^{k} x_i \leq \frac{1}{1 - 2\sin\frac{\alpha}{2}}\|v_0 - v_k\|$. Since $x_i + x_{i+1} > r$ for any $1 \leq i < k$, we have $k \leq 2\lceil \frac{\frac{\|v_0 - v_k\|}{1 - 2\sin\frac{\alpha}{2}}}{r} \rceil \leq 2\lceil \frac{1}{1 - 2\sin\frac{\alpha}{2}} \rceil \cdot \lceil \frac{\|v_0 - v_k\|}{r} \rceil$. Thus, the total energy consumption of the found path is $\sum_{i=1}^{k} \mathbf{c}(x_i) \leq k\mathbf{c}(r)$. This implies that the LEARN routing protocol finds the path whose energy consumption is at most $2\lceil \frac{1}{1 - 2\sin\frac{\alpha}{2}} \rceil$ times of the optimum. This completes the proof. ■

Notice that the above theorem applies to *any* general energy model $\mathbf{c}(\|uv\|)$, where $\mathbf{c}(x)$ is a non-decreasing function on x, and $\frac{x}{\mathbf{c}(x)}$ is an increasing function when $x < \mathbf{r}_0$.

We then prove the energy efficiency of LEARN routing protocol for the case when $0 < \mathbf{r}_0 < r$. See Figure 3(b) for illustration. In this case, we need an additional requirement that the energy cost $\mathbf{c}(x)$ is *smooth* around $\mathbf{r}_0$, *i.e.*, there exists a function $f()$ such that $\mathbf{c}(a\mathbf{r}_0) \leq f(a) \cdot \mathbf{c}(\mathbf{r}_0)$ for any constant a in certain range defined by $\eta_1 \leq a \leq \eta_2$.

Theorem 4: When LEARN routing indeed finds a path $\mathbf{P}_{LEARN}(s,t)$ from the source s to the target t and $\mathbf{r}_0 < r$, the total energy consumption of the found path is within a constant factor of the optimum.

Proof: Notice that when $\mathbf{r}_0 < r$, our LEARN routing protocol will select a forwarding neighbor v for an intermediate node u such that $\|uv\| \leq \eta_2\mathbf{r}_0$ and $|\|uv\| - \mathbf{r}_0|$ is minimized. Consider any path $v_0 v_1 v_2 \cdots v_{k-1} v_k$ found by our routing protocol to connect source node $s = v_0$ and destination node $t = v_k$. Let $x_i = \|v_{i-1}v_i\|$. Notice that since $\mathbf{r}_0 < r$, then for any path connecting v_0 and v_k, its energy consumption is at least $\frac{\|v_0 - v_k\|}{\eta_2 \mathbf{r}_0}\mathbf{c}(\mathbf{r}_0)$. We will show that power consumption of the path found by LEARN is within a constant factor of this.

When $\eta_1\mathbf{r}_0 \leq x_i \leq \eta_2\mathbf{r}_0$ for $1 \leq i \leq k$, the total energy consumption is $\sum_{i=1}^{k} \mathbf{c}(x_i) \leq f(\eta_2) \cdot k \cdot \mathbf{c}(\mathbf{r}_0)$. Notice that $\sum_{i=1}^{k} x_i \leq \frac{1}{1 - 2\sin\frac{\alpha}{2}}\|v_0 - v_k\|$ (by Theorem 1) implies $k \leq \frac{\|v_0 - v_k\|}{(1 - 2\sin\frac{\alpha}{2})\eta_1\mathbf{r}_0}$. Then ratio of the energy consumed by this path over the optimum energy consumption is at most $\frac{f(\eta_2) \cdot k \cdot \mathbf{c}(\mathbf{r}_0)}{\frac{\|v_0 - v_k\|}{\eta_2 \mathbf{r}_0}\mathbf{c}(\mathbf{r}_0)} \leq \frac{\eta_2 f(\eta_2)}{\eta_1(1 - 2\sin\frac{\alpha}{2})}$ (which is a constant). This completes the proof. ■

For example, for the energy model $\mathbf{c}(x) = x^2 + c$ with $\eta_1 = 1/2$ and $\eta_2 = 2$, we have $f(a) = (a^2 + 1)/2$ and $f(\eta_2) = 5/2$. Thus, our routing algorithm finds a path whose energy consumption is at most $\frac{10}{1 - 2\sin\frac{\alpha}{2}}$ times of optimum when $\mathbf{r}_0 < r$.

V. Critical Transmission Radius of LEARN

This section is devoted to study the critical transmission radius for LEARN routing method in random networks. In any greedy routing method (including LEARN), the packet may be dropped by some intermediate node u before it reaches the destination t when node u could not find any of its neighbors that is "better" than itself. Thus, to ensure that the routing is successful for every pair of possible source and destination nodes, each node in the network should have a sufficiently large transmission radius such that each intermediate node u will always find a better neighbor.

A. Critical Transmission Radius

Assume that V is the set of all wireless nodes and each node has a transmission radius r. Then the physical communication network is modeled by a unit disk graph $G(V, r)$, where two nodes u and v are connected in $G(V, r)$ if and only if their Euclidean distance is at most r. A routing method $\mathcal{A}$ is *successful* over a network G if the routing method $\mathcal{A}$ can find a path for any pair of source and destination nodes. Then we can define the critical transmission radius of $\mathcal{A}$ as follows:

Definition 4: Given a routing method $\mathcal{A}$ and a set of wireless nodes V, the *critical transmission radius*, denoted as $\rho_{\mathcal{A}}(V)$, for successful routing of $\mathcal{A}$ over V is the *minimum* transmission radius r such that the routing method $\mathcal{A}$ over the network $G(V, r)$ is successful.
The subscript $\mathcal{A}$ will be omitted from $\rho_{\mathcal{A}}(V)$ if it is clear from the context.

Previously, several studies (*e.g.* [18]–[20]) focused on the critical transmission radius for certain network properties such as connectivity, k-connectivity, and coverage. However, there is not much study for the critical transmission radius for certain routing methods, except an early result [21] for various transmission protocols (slotted ALOHA and CSMA) and very recent result [22] for traditional greedy routing. Obviously, for traditional greedy routing, which selects a neighbor v of u that is closest to the destination node t, and $\|v - t\| < \|u - t\|$, the critical transmission radius $\rho(V)$ for successful routing is $\max_{u,v} \min_{w \in \mathbf{L}(u,v)} \|w - u\|$ where lune $\mathbf{L}(u, v)$ is the intersection of two disks centered at u and v respectively using $\|u - v\|$ as radius. It was proved in [22] that for any constant $\varepsilon > 0$, it is *a.a.s.* that

$$(1 - \varepsilon)\sqrt{\frac{\beta_0 \ln n}{\pi n}} \leq \rho(\mathcal{P}_n) \leq (1 + \varepsilon)\sqrt{\frac{\beta_0 \ln n}{\pi n}},$$

where $\beta_0 = 1/(\frac{2}{3} - \frac{\sqrt{3}}{2\pi}) \simeq 1.6^2$ and $\mathcal{P}_n$ is a Poisson point process of density n over a convex compact region Ω with unit area and bounded curvatures.

B. Critical Transmission Radius of LEARN

It is easy to show that, given a set of nodes V already distributed in a region Ω, the critical transmission radius $\rho(V)$ for successful routing by our restricted greedy routing LEARN is

$$\max_{u,v} \min_{w:\ \angle wuv \leq \alpha} \|w - u\| \tag{1}$$

where α is the parameter used by LEARN. By setting the $r = \rho(V)$, LEARN can always find a forwarding node inside the restricted sector region, thus can guarantee its packet delivery. When the destination node is fixed, say node t, the critical transmission radius will be $\max_u \min_{w:\ \angle wut \leq \alpha} \|w - t\|$.

We assume that the network nodes are given by a Poisson point process $\mathcal{P}_n$ of density n over a convex compact region Ω with unit area and bounded curvatures. Then we can prove a similar result as in [22] for our restricted greedy routing method LEARN. The following two theorems provide the upper and lower bounds of the critical transmission radius of LEARN routing. Due to space limitation, detailed proofs of these two theorems are provided in *Supplemental Material*.

Theorem 5: The LEARN routing (with parameter $\alpha < \frac{\pi}{3}$) will find a path from the source to the target *asymptotically almost surely* when the transmission radius r satisfies $n\pi r^2 = \beta \ln n$ for any constant $\beta > \beta_0 = \frac{\pi}{\alpha}$.

Theorem 6: The LEARN routing (with parameter $\alpha < \frac{\pi}{3}$) will *not* be able to find a path from the source to the target *asymptotically almost surely* when the transmission radius r satisfies $n\pi r^2 = \beta \ln n$ for any constant $\beta < \beta_0 = \frac{\pi}{\alpha}$.

Notice that above results assume that the deployment region Ω has a unit area. Generally, we will often have a convex and compact region Ω with area $\mathcal{D}$, and the transmission radius r could be fixed (or dynamically changed based on node density). Assume again that the network nodes are produced by a Poisson process with rate n (*i.e.*, the expected number of nodes in a unit area is n, thus the total number of deployed nodes in the area is $n\mathcal{D}$). Then by a proper scaling of the distance unit, we have the following theorem.

Theorem 7: When the transmission radius r and the Poisson process rate n satisfy that $n\pi r^2 = \beta \cdot \ln(\mathcal{D} \cdot n)$ for any $\beta > \beta_0$, our LEARN routing protocol will successfully route the data *a.a.s.*. When $n\pi r^2 = \beta \cdot \ln(\mathcal{D} \cdot n)$ for any $\beta < \beta_0$, our LEARN routing protocol will *not* be able to route the data *a.a.s.*.

C. General Results on Critical Transmission Radius

So far, we mainly concentrated on the routing LEARN. Notice that the critical transmission radius of our LEARN-G routing protocol will be exactly same as the traditional greedy routing [9] method since at last we use the greedy routing to find the forwarding node if LEARN fails. There are a number of other localized routing methods developed already and many to be developed in the future. We thus would like to know the general critical transmission radius for successful routing by any localized routing method $\mathcal{A}$. We then generalize the above theorems to the following theorem.

Theorem 8: For a general localized routing method $\mathcal{A}$, the critical transmission radius is $\rho_{\mathcal{A}}(\mathcal{P}_n) = \sqrt{\frac{\beta_{\mathcal{A}} \ln(\mathcal{D} \cdot n)}{n\pi}}$. Here $\beta_{\mathcal{A}}$ is the ratio of the area of the disk centered at an intermediate node u with radius r over the area of the *forwarding region* in this disk from where the intermediate node u can choose its next neighbor w; $\mathcal{D}$ is the area of the convex and compact deployment region; and n the rate of the Poisson point process. We require that the forwarding region of any intermediate node u for any target node t is *convex and compact* and at least a constant fraction of the forwarding region is contained inside the deployment region Ω.

Notice that the above theorem not only applies to the routing method, it also applies to the critical radius for the connectivity of the network in which $\beta_{\mathcal{A}} = 1$. This is based on the following observation: a network, formed by a set V of n nodes and each node has a transmission radius r, is connected if and only if the routing method $\mathcal{H}$ that uses the path with the minimum hop number can successfully find a path for every pair of source and destination nodes. For this special routing method $\mathcal{H}$, clearly the area to find the forwarding node w by a node u is the disk $\mathbf{D}(u, r)$, *i.e.*, $\beta_{\mathcal{H}} = 1$.

So far, we assumed that the link is always reliable and the nodes are always awake. This is always an ideal case. To capture the practical aspects of wireless networks, we assume that a wireless link uv is reliable with a *constant* probability $p_1 > 0$ and each node is awake with a *constant* probability $p_2 > 0$. Similarly we can show that that the critical transmission radius for a successful routing by a general routing method $\mathcal{A}$ is $\rho_{\mathcal{A}}(\mathcal{P}_n) = \sqrt{\frac{\beta_{\mathcal{A}} \ln(\mathcal{D} \cdot n)}{n\pi p_1 p_2}}$.

VI. 3D LEARN Routing in 3D Wireless Networks

Recently, three-dimensional (3D) wireless network has received significant attention [23]–[25], due to its wide range of potential applications (such as underwater sensor networks). The design of networking protocols for 3D wireless networks is surprisingly more difficult than that for 2D networks. For example, to guarantee *packet delivery* of 2D localized routing, face routing [9] can be used on planar topology to recovery from local minimum. However, there is no planar topology concept any more in 3D networks. In fact, Durocher *et al.* [26] recently proved that there is *no* deterministic localized routing algorithm for 3D networks that guarantees the delivery of packets. For energy efficiency, recently, Flury and Wattenhofer [27] also showed an example of a 3D network (Figure 1 of [27]) where the path found by *any* localized routing protocol to connect two nodes s and t has energy consumption asymptotically at least $\Theta(OPT^3)$ in the worst case. Therefore, we are also interested in extending our proposed LEARN routing into a 3D energy efficient routing.

Fortunately, as the classical greedy routing, LEARN routing protocol can be directly applied in 3D networks. The restricted region now is a 3D cone defined by parameter α. The only difference is that face routing cannot be extended in 3D to guarantee packet delivery anymore. As suggested in [27], randomized method may be used as the backup method. The path efficiency of 3D LEARN routing is straightforward. The proofs of Theorem 1, Theorem 3, and Theorem 4 do not need any changes in 3D case. However, the critical transmission radius of 3D LEARN is different with 2D's one. By definition, we would like to find $\rho(V)$ such that by setting $r = \rho(V)$ 3D LEARN routing can always find a forwarding node inside the 3D cone region.

Generally, we assume that the network is distributed in a convex and compact 3D region Ω with volume $\mathcal{D}$, and the transmission radius is r. The network nodes are produced by a Poisson process with rate n (*i.e.*, the expected number of nodes in a unit-volume area is n). Using the similar technique in Section V, we can prove the following theorem for 3D LEARN routing.

Theorem 9: When the transmission radius r and the Poisson process rate n satisfy that $\frac{4}{3}n\pi r^3 = \beta \cdot \ln(\mathcal{D} \cdot n)$ for any $\beta > \beta_3$, our LEARN routing protocol will successfully route the data *a.a.s.*. When $\frac{4}{3}n\pi r^3 = \beta \cdot \ln(\mathcal{D} \cdot n)$ for any $\beta < \beta_3$, our LEARN routing protocol will *not* be able to route the data *a.a.s.*. Here, $\beta_3 = \frac{2}{1-cos\alpha}$.

Due to space limit, we ignore the detail proof of the theorem. However, the basic idea is similar to the proofs of Theorem 5 and Theorem 6 as provided in *Supplemental Material*. Notice that $\beta_3 = \frac{4\pi/3}{2\pi(1-cos\alpha)/3} = \frac{2}{1-cos\alpha}$ is the ratio between the volume of a unit ball and the volume of a 3D cone (the forwarding region) inside the ball. Similar with Theorem 8, we can prove the following theorem for general 3D localized routing.

Theorem 10: For a general 3D localized routing method $\mathcal{A}$, the critical transmission radius is $\rho_{\mathcal{A}}(\mathcal{P}_n) = \sqrt[3]{\frac{3\beta_{\mathcal{A}} \ln(\mathcal{D} \cdot n)}{4n\pi}}$ where $\beta_{\mathcal{A}}$ is the ratio of the volume of the neighborhood and the volume of the forwarding region.

VII. Related Work

A. Localized Routing

The geometric nature of the multihop wireless networks allows the promising idea: localized routing protocols. The most popular localized routing is *greedy routing* where the current node always finds the next relay node who is the nearest to the destination. Though greedy routing (or its variation) was widely used, it is easy to construct an example where greedy routing will not succeed to reach the destination but fall into a local minimum. To guarantee the packet delivery, *face routing* is proposed by [28]. Its idea is to walk along the faces (using the right hand rule to explore each face) which are intersected by the line segment between the source and the destination. Kranakis *et al.* [28] and Kuhn *et al.* [14] proved that face routing guarantees to reach the destination after traversing at most $O(n)$ edges when the network topology is a planar graph. Though face routing terminates in linear time, it is not satisfactory, since already a simple flooding algorithm can terminate in $O(n)$ steps. Then several refined methods [11], [14] have been proposed to avoid exploring the complete boundary of faces by using restricted search areas.

Greedy routing is simple and efficient but cannot guarantee the packet delivery, while face routing can guarantee the delivery but may take a lengthy exploration. One natural improvement is to combine greedy routing and face routing by using face routing to recover the route after greedy method fails in local minimum. Many routing protocols [7], [9]–[11] used this approach, such as *greedy face routing* (GFG) [9].

Although face routing or greedy face routing can guarantee the packet delivery on planar networks and some localized routing protocols can guarantee the delivery if certain geometry structures are used as the routing topology, none of them guarantees the ratio of the distance traveled by the packets over the minimum possible. Bose and Morrin [29] proposed a method to bound this ratio using the Delaunay triangulation. They showed that the distance traveled by the packet is within a constant factor of the distance between the source and the destination. Since constructing Delaunay triangulation in a distributed manner is communication expensive, Wang and Li [30] proposed using local Delaunay triangulation instead of Delaunay triangulation. However, they also showed that the Delaunay based routing [29], [30] is not energy efficient (*i.e.*, the power spent by the route could be sufficiently larger than the optimal least-power path) even though the distance traveled is bounded by constant with the optimal.

B. Energy Efficient Routing

Since energy is a scarce resource which limits the life of wireless network, a number of energy efficient routing protocols [1]–[8] have been proposed recently using a variety techniques. Classical routing algorithm may be adapted to take into account energy-related criteria rather than classical metrics such as delay or hop distance. Most of the proposed energy-aware metrics are defined as a function of the energy required to communicate on a link [1], [4] or a function of the nodes remaining lifetime [2]. However, to minimize the global consumed energy of selected route, most of minimum energy routing algorithms [2]–[4] are centralized algorithms. In this paper, we focus on stateless localized routing. Thus, we only review the following related work about energy efficient techniques for *localized* routing which address how to save energy when making local routing decision.

Melodia *et al.* [5] proposed a *partial topology knowledge forwarding* for sensor network, where each node selects the shortest energy-weighted path based on local knowledge of topology. They assumed the neighborhood discovery protocol provides each node the local knowledge of topology within certain range. They gave a linear programming formulation to select the range which minimizes the energy expenditure of the network. Since the solution of the linear programming problem is not feasible in practice, they also proposed a distributed protocol to adjust the topology knowledge range.

Stojmenovic and Lin [7] proposed a power-aware localized routing which combining the cost metric based on remaining battery power at nodes and the power metric based on the transmission power related to distance between nodes. They proved the loop-free properties of their methods and show their efficiency by experiments. Stojmenovic and Datta [6] further combined the above method with face routing to guarantee the delivery. However, they provide no theoretical guarantee of energy efficiency for all of their methods.

Kuruvila *et al.* [8] also proposed a new power-aware localized routing *power progress routing* in which the current node select its neighbor who minimizes the ratio between the energy consumption and the distance progress towards the destination as the reply node. This is the first localized method based on the *cost to progress ratio*. The authors also proposed some variants and conducted extensive simulations. Stojmenovic [16] then generalized this concept to *cost to the progress ratio*, and provided a general routing framework and a nice survey on many similar routing methods. The usage of energy millage in our routing method to bound the energy efficiency is inspired by these study.

Seada *et al.* [13] proposed a power-aware greedy routing method which not only consider the transmission power but also the reliability of each link. Their work focuses on routing in sensor networks with lossy links. They used the product of packet reception rate and distance improvement as a new local metric for greedy routing. They showed that it can significantly enhance the delivery rate and energy efficiency in lossy sensor networks. Notice that they considered a fix transmission power, which is different with our model.

Li *et al.* [12] considered the joint localized routing and power control problem also under loss links. Using more realistic models for wireless channel fading as well as radio modulation and encoding, they derived the optimal power control strategy over a given link. They then proposed a local power-efficiency metric for localized routing which is very similar to our energy millage. By considering link reliability, their metric is the ratio between the product of packet reception rate and distance improvement and the transmission power. Via simulations, they show that their method can have close performance compared with the global optimal. However, no theoretical guarantee is proved.

VIII. Conclusion

We proposed the localized energy aware restricted neighborhood routing protocol (LEARN) for wireless networks. We theoretically proved that our LEARN routing protocol is energy efficient if it can find a path. We also studied its critical transmission radius for the successful packet delivery *a.a.s.*. We also extended the proposed routing method into 3D networks. Our mathematical formulation also extends to any routing protocol in which the region to find the next hop node by an intermediate node is compact and convex. We conducted extensive simulations to study the performance of our LEARN routing. Detailed simulation results on its critical transmission radius and network performances are provided in in *Supplemental Material*. To the best of our knowledge, our new localized routing method is the first localized routing protocol that has theoretical guarantee of energy efficiency of the generated routes in random networks with high probability.

Notice that our theoretical studies assumed a static random network. For static networks, greedy routing will terminate after a finite time because at any intermediate node u, it always makes a positive movement towards the target t if any intermediate node u finds a forwarding node. However, this argument will not extend directly to the case when the intermediate nodes are mobile. Therefore, further study on the performance analysis of LEARN in mobile networks is left as future work. It is still possible to prove that LEARN routing protocol will terminate in mobile networks with high probability under certain assumptions. However, this does not directly implies that it will find the path from any pair of nodes asymptotically almost surely. We leave it as a future work to compute the critical transmission radius for mobile networks.

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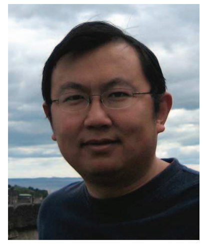

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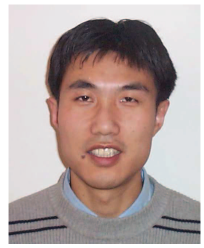

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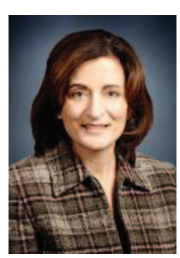

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