Localized Topology Control for Unicast and Broadcast in Wireless Ad Hoc Networks

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Abstract

We propose a novel localized topology control algorithm for each wireless node to locally select communication neighbors and adjust its transmission power accordingly, such that all nodes together self-form a topology that is energy efficient simultaneously for both unicast and broadcast communications. We theoretically prove that the proposed topology is *planar*, which guarantees packet delivery if a certain localized routing method is used; it is power efficient for unicast– the energy needed to connect any pair of nodes is within a small constant factor of the minimum; it is also asymptotically optimum for broadcast: the energy consumption for broadcasting data on top of it is asymptotically the best among all structures constructed using only local information; it has a constant bounded logical degree, which will potentially save cost of updating routing table if used. We further prove that the expected average physical degree of all nodes is a small constant. To the best of our knowledge, this is the *first* localized algorithm to build a structure with all these desirable properties. Previously, only a centralized algorithm was reported in [3]. Moreover, by assuming that the node ID and its position can be represented in $O(\log n)$ bits for a wireless network of n nodes, the total number of messages by our methods is in the range of $[5n, 13n]$, where each message is $O(\log n)$ bits. Our theoretical results are corroborated in the simulations.

Keywords

Graph theory, localized algorithm, wireless ad hoc networks, topology control, power efficient, low weight, low interference, unicast, broadcast.

I. Introduction

A wireless *ad hoc* network consists of a distribution of radios in certain geographical area. Unlike cellular wireless networks, there is no centralized control in the network, and wireless devices (called *nodes* hereafter) can communicate via multi-hop wireless channels: a node can reach all nodes inside its transmission region while two far-away nodes communicate through the relaying by intermediate nodes. An important requirement of these networks is that they should be self-organizing, *i.e.*, transmission ranges and data paths are dynamically restructured with changing topology. Energy conservation and network performance are probably the most critical issues in wireless *ad hoc* networks, because wireless devices are usually powered by batteries only and have limited computing capability and memory.

A wireless *ad hoc* or sensor network is modelled by a set V of n wireless nodes distributed in a two-dimensional plane. Each node has the same *maximum* transmission range R. By a proper scaling, we assume that all nodes have the maximum transmission range equal to one unit. These wireless nodes define a *unit disk graph* $UDG(V)$ in which there is an edge between two nodes iff the Euclidean distance between them is at most one unit. In other words, we assume that two nodes can always receive the signal from each other directly if the Euclidean distance between them is no more than one unit. Notice that, in practice, the transmission region of a node is not necessarily a perfect *disk*. As done by most results in the literature, for simplicity, we model it by *disk* in order to first explore the underlying nature of ad hoc networks. Hereafter, $UDG(V)$ is always assumed to be connected. We also assume that all wireless nodes have distinctive identities(IDs) and each wireless node knows its position information. More specifically, it is enough in our protocol if each node knows the relative position of its one-hop neighbors. The relative position of neighbors can be estimated by the *direction of signal arrival* and the *strength of signal*. The geometry location of a wireless node can also be obtained by a localization method, such as [27], [7], [13]. We adopt the most common power-attenuation model from literature: the power needed to support a link uv is assumed to be $\|uv\|^\beta$, where $\|uv\|$ is the Euclidean distance between u and v, β is a real constant between 2 and 5 depending on the wireless transmission environment. Note that in practice, the receiving node v will consume power to receive the signal and the transmitting node u will spend power to prepare the signal. In this paper, we mainly consider the transmission power proportional to $\|uv\|^\beta$.

The localized *topology control* technique lets each wireless device *locally* adjust its transmission range and select certain neighbors for communication, while maintaining a decent global structure to support energy efficient routing and to improve the overall network performance. By enabling each wireless node to shrink its transmission power (which could be much smaller than its maximum transmission power) sufficient enough to cover its farthest selected neighbor in routing, topology control schemes can not only save energy and prolong network life, but also can improve network throughput through mitigating the MAC-level medium contention by using possibly shorter links. Unlike traditional wired and cellular networks, the movement of wireless devices during the communication could change the network topology in some extent. Hence, it is more challenging to design a topology control algorithm for *ad hoc* wireless networks: the topology should be locally and self-adaptively maintained with low communication cost, without affecting the whole network.

The main contributions of this paper are as follows. We present the *first* localized algorithm to construct a *unified* energy-efficient topology for unicast and broadcast in wireless ad hoc/sensor networks. In one single structure, we guarantee the following network properties:

1. **power efficient unicast**: given any two nodes, there is a path connecting them in the structure with total power cost no more than $2\rho + 1$ times of the power cost of any path connecting them in the original network. Here $\rho > 1$ is some constant that will be specified later in our algorithm. We assume that each node u can adjust its power sufficiently to cover its next-hop v on any selected path for unicast.

2. **power efficient broadcast**: the power consumption for broadcast is within a constant factor of optimum among all *locally* constructed structures. To prove this, we essentially prove that the structure is *low-weighted*: its total edge length is within a constant factor of that of Euclidean Minimum Spanning Tree (EMST). For broadcast or generally multicast, we assume that each node u can adjust its power sufficiently to cover its farthest downstream node on any selected structure (typically a tree) for multicast.

3. **bounded logical node degree**: each node has to communicate with at most $k - 1$ logical neighbors, where $k \geq 9$ is an adjustable parameter.

4. **bounded average physical node degree**: the expected average physical node degree is at most a small constant. Here the physical degree of a node u in a structure H is defined as the number of nodes inside the disk centered at u with radius $\max_{uv \in H} \|uv\|$.

5. **planar**: there are no edges crossing each other. This enables several localized routing algorithms, such as [2], [16], [21], [22], to be performed on top of this structure and guarantee the packet delivery without using the routing table.

6. **neighbors Θ-separated**: the directions between any two logical neighbors of any node are separated by at least an angle θ, which as we will see reduces the signal interference.

In graph theoretical terminologies, given a unit disk graph modelling the wireless ad hoc networks, we propose a localized method to build a low-weighted planar power-spanner with a bounded logical node degree. Here a geometric structure is called *low-weighted* if its total edge length is no more than a small constant factor of that of the Euclidean minimum spanning tree. To the best of our knowledge, it is the *first* known *localized* algorithm to construct such a *single* structure with all these desired properties. Previously, only a centralized algorithm was reported in [3]. Moreover, by assuming that the node ID and its position can be represented in $O(\log n)$ bits each for a wireless network of n nodes, we show that the structure can be initially constructed using $5n$ to $13n$ messages.

In addition, we prove that the expected average node interference in the structure is bounded by a small constant. This is significant in its own due to the following reasons: it has been taken for granted that "*a network topology with small logical node degree will guarantee a small interference*" and recently Burkhart *et al.* [4] showed that this is not true generally. Our results show that, although generally a small logical node degree cannot guarantee a small interference, the expected average interference is indeed small if the logical communication neighbors are chosen carefully. All our theoretical results are corroborated in simulations.

We also show that our structure can be easily updated in a dynamic environment when node moves or dies after the battery power is drained. When a node moves, the topology can be locally and dynamically self-maintained without affecting the whole network, since each node adjusts its transmission range and selects neighbors only according to its neighbor information.

To facilitate the localized construction of such a unified energy-efficient topology, in the paper, we will first give an improved method to construct degree-bounded planar spanner by using relative positions only. The new structure has the same power spanning ratio $\rho = \frac{\sqrt{2}^{\beta}}{1-(2\sqrt{2}\sin\frac{\pi}{k})^{\beta}}$ as the structure proposed in [34]. Here $k \geq 9$ is a customizable parameter. In addition, the directions between any two neighbors of each node are separated by at least a certain angle θ depending on the parameter k. Simulations show that the node interference in our new structure is indeed smaller than the structure proposed in [34].

The rest of the paper is organized as follows. In Section II, we review some prior arts in topology control, and summarize some preferred properties of network topology for unicast and broadcast. Section III presents an improved algorithm based on [34] to build a degree-bounded planar spanner with Θ-separated property. We then propose, in Section

IV, the first localized algorithm to construct planar spanner with bounded-degree and low weight. We study the expected interference of various structures in Section V. In Section VI, we conduct extensive simulations to validate our theoretical results. Finally, we conclude our paper in Section VII.

II. Current State of Knowledge

A. Energy-Efficient Unicast Topology

Several structures have been proposed for topology control in wireless ad hoc networks. The *relative neighborhood graph*, denoted by $RNG(V)$ [35], consists of all edges uv such that the intersection of two circles centered at u and v and with radius $\|uv\|$ do not contain any vertex w from the set V. The *Gabriel graph* [11] $GG(V)$ contains an edge uv if and only if $disk(u,v)$ contains no other points of V, where $disk(u,v)$ is the disk with edge uv as a diameter. For convenience, also denote GG and RNG as the intersection of $GG(V)$ and $RNG(V)$ with $UDG(V)$ respectively. Both GG and RNG planar. They are connected, and contain the *Euclidean minimum spanning tree*$(EMST)$ of V if UDG is connected. RNG is not power efficient for unicast, since the power stretch factor of RNG is $n-1$. Both RNG and GG are not degree-bounded. The *Yao graph* [42] with an integer parameter $k > 6$, denoted by $\overrightarrow{YG}_k$, is defined as follows. At each node u, any k equally-separated rays originated at u define k cones. In each cone, choose the shortest edge $uv \in UDG(V)$ among all edges emanated from u, if there is any, and add a directed link $\overrightarrow{uv}$. Ties are broken arbitrarily or by ID. The resulting directed graph is called the *Yao graph*. It is well-known that the Yao structure is power efficient for unicast. Several variations [25] of the Yao structure could have bounded logical node degree also. However, all Yao related structures are not planar graph.

Li *et al.* [23] proposed the Cone Based Topology Control (CBTC) algorithm to first focus on several desirable properties, in particular being an energy spanner with bounded degree. It is basically similar to the Yao structure for topology control. Each node u finds a power $p_{u,\alpha}$ such that in every cone of degree α surrounding u, there is some node that u can reach with power $p_{u,\alpha}$. Here, nevertheless, we assume that there is a node reachable from u by the maximum power in that cone. Notice that the number of cones to be considered in the traditional Yao structure is a constant k. However, unlike the Yao structure, for each node u, the number of cones needed to be considered in the method proposed in [23] is about $2n$, where each node v could contribute two cones on both side of segment uv. Then the graph G_α contains all edges uv such that u can communicate with v using power $p_{u,\alpha}$. They proved that, if $\alpha \le \frac{5\pi}{6}$ and the UDG is connected, then graph G_α is a connected graph. On the other hand, if $\alpha > \frac{5\pi}{6}$, they showed that the connectivity of G_α is not guaranteed by giving some counter-example [23]. Unlike the Yao structure, the final topology G_α is not necessarily a bounded degree graph.

Bose *et al.* [3] proposed a centralized method with running time $O(n \log n)$ to build a degree-bounded planar spanner for a two-dimensional point set. It constructs a planar t-spanner with low-weight for a given nodes set V, for $t = (1 + \pi) \cdot C_{del} \simeq 10.02$, such that the node degree is bounded from above by 27. Hereafter, we use C_{del} to denote the spanning ratio of the Delaunay triangulation [10], [18], [17]. However, the distributed implementation of this centralized method takes $O(n^2)$ communications in the worst case for a set V of n nodes.

Wang and Li [38] proposed the first efficient localized algorithm to build a degree-bounded planar spanner BPS for wireless ad hoc networks. Though their method can achieve three desirable features: planar, degree-bounded, and power efficient, the theoretical bound on the node degree of their structure is a large constant. For example, when $\alpha = \pi/6$, the theoretical bound on node degree is 25. In addition, the communication cost of their method can be very high, although it is $O(n)$ theoretically, which is achieved by applying the method in [5] to collect 2-hop neighbors information. The hidden constant is large: it is several hundreds.

Recently, Song *et al.* [34] proposed two methods to construct degree-bounded power spanner, by applying the ordered Yao structures on Gabriel graph. They achieved better performance with much lower communication cost, compared with the method in [38]. One method in [34] only costs $3n$ messages for the construction, and guarantees that there is at most one neighbor node in each of the $k = 9$ equal-sized cones.

Notice that the structures constructed by the methods proposed in [38], [34] are not guaranteed to be low-weighted. Both structures are planar and degree-bounded. The structure constructed in [34] is only a power-spanner, while the structure constructed in [38] is also a length-spanner. Notice that it is known that a length-spanner is always a power spanner [25]. The main contribution of this paper is that we propose the *first* method to locally construct a topology that is planar, length-spanner, bounded-degree, and low-weighted.

In summary, for energy efficient unicast routing, the topology is preferred to have following features:

1. Power Spanner: Formally speaking, a subgraph H is called a *power spanner* of a graph G if there is a positive real constant ρ such that for any two nodes, the power consumption of the shortest path in H is at most ρ times of the power consumption of the shortest path in G. Here ρ is called the *power stretch factor* or *spanning ratio*.
2. Degree Bounded: It is also desirable that the logical node degree in the constructed topology is bounded from above by a small constant. Bounded logical degree structures find applications in Bluetooth wireless networks since a *master* node can have only 7 active slaves simultaneously. A structure with small logical node degree will save the cost of updating the routing table when nodes are mobile. A structure with a small degree and using shorter links could improve the overall network throughout [20].
3. Planar: A network topology is also preferred to be planar (no two edges crossing each other in the graph) to enable some localized routing algorithms work correctly and efficiently, such as *Greedy Face Routing* (GFG) [2], *Greedy Perimeter Stateless Routing* (GPSR) [16], *Adaptive Face Routing*(AFR) [21], and *Greedy Other Adaptive Face Routing* (GOAFR) [22]. Notice that with planar network topology as the underlying routing structure, these localized routing protocols guarantee the message delivery without using a routing table: each intermediate node can decide which logical neighboring node to forward the packet using only local information and the position of the source and the destination.

B. Energy-Efficient Broadcast Topology

Broadcast is also a very important operation in wireless ad hoc networks, as it provides an efficient way of communication that does not require global information and functions well with topology changes. For example, many unicast routing protocols [15], [28], [31],

[30], [33] for wireless multi-hop networks use broadcast in the stage of route discovery. Similarly, several information dissemination protocols in wireless sensor networks use some forms of broadcast/multicast for solicitation or collection of sensor information [12], [14], [43]. Since sensor networks mainly [1] use broadcast for communication, how to deliver messages to all the wireless devices in a scalable and power-efficient manner has drawn more and more attention. Not until recently have research efforts been made to devise power-efficient broadcast structures for wireless ad hoc networks.

Notice that, a broadcast routing protocol can be interpreted as *flood-based* broadcasting on a subgraph of original communication networks, since *any* broadcast routing is viewed as an arborescence (a directed tree) T, rooted at the source node of the broadcasting, that spans all nodes. Once the structure is constructed, the broadcast is a simple flooding: once a node got the broadcast message from its *logical* neighbors for the first time, it will simply forward it to all its *logical* neighbors either through one-to-one or one-to-all communications. Let $f_T(p)$ denote the transmission power of the node p required by broadcasting message on top of the tree T. We assume that the tree T is a directed graph rooted at the source of the broadcasting session: link $pq \in T$ denotes that node p forwarded message to node q. For any leaf node p of T, clearly we have $f_T(p) = 0$ since it does not have to forward the data to any other node. For any internal node p of T,

$$f_T(p) = \max_{pq \in T} \|pq\|^{\beta}$$

under our energy model if an one-to-all communication model is used; and

$$f_T(p) = \sum_{pq \in T} \|pq\|^{\beta}$$

under our energy model if an one-to-one communication model is used. In the literature, the one-to-all communication model (a node p transmits once at power $\max_{pq \in T} \|pq\|^{\beta}$ and all its downstream nodes get the data) is typically assumed. The total energy required by T is $\sum_{p \in V} f_T(p)$.

Minimum-energy broadcast routing (MEB) in a simple ad hoc networking environment has been addressed in [8], [19], [40]. It is known [8] that the MEB problem is NP-hard, *i.e.*, it cannot be solved in polynomial time unless P=NP. Three greedy heuristics were proposed in [40] for the MEB problem: EMST (minimum spanning tree), SPT (shortest-path tree), and BIP (broadcasting incremental power). Wan *et al.* [36], [37] showed that the approximation ratios of EMST and BIP are at most 12; on the other hand, the approximation ratio of SPT is at least $\frac{n}{2}$, where n is the number of nodes. Unfortunately, none of the above structures can be formed and updated locally.

RNG, which can be constructed locally, has been used for broadcasting in wireless ad hoc networks [32]. However, an example was given in [24] to show that the total energy used by broadcasting on RNG could be about $O(n^\beta)$ times of the minimum. Several localized broadcasting protocols [41], [6] are proposed recently, however, all of them did not provide their theoretical performance bound. In fact, Li [24] showed that, there is *no* deterministic localized algorithm to find a structure that approximates the total energy consumption of broadcasting within a constant factor of the optimum. Furthermore, in the worst case, the energy cost for broadcasting on *any* locally constructed and connected structure is at least $\Theta(n^{\beta-1})$ times the optimum for a network of n nodes. On the other hand, given any low-weighted structure H, *i.e.*, $\omega(H) \leq O(1) \cdot \omega(EMST)$, they proved the following lemma

Lemma 1: [24] $\omega_\beta(H) \leq O(n^{\beta-1}) \cdot \omega_\beta(EMST)$, where H is any low-weighted structure. Here $\omega(G)$ is the total length of the links in G, *i.e.*, $\omega(G) = \sum_{uv \in G} \|uv\|$, and $\omega_\beta(G)$ is the total power consumption of links in G, *i.e.*, $\omega_\beta(G) = \sum_{uv \in G} \|uv\|^\beta$. Consequently, low-weighted structure is asymptotically optimal for broadcasting among any connected structures built in a localized manner. Notice that, the above analysis is based on the assumption that every link is used during the broadcast (one-to-one communication), such as using the TDMA scheme. Even considering one-to-all communication (*i.e.*, the broadcast signal sent by a node can be received by all nodes in its transmission region simultaneously), the above claim is also correct. The reason is basically as follows. Let $B_s(H)$ be the total energy consumed by broadcasting on a structure H with sender s using the one-to-all communication model. Clearly, ***any*** flood-based broadcast based on a structure H consumes energy at most $\sum_{e_i \in H} e_i^\beta$ if the message received by an intermediate node v is not forwarded to its parent, *i.e.*, the node that just forwarded this message to v; and the total energy is at most $2\sum_{e_i \in H} e_i^\beta$ if an intermediate node v blindly forward the data (*i.e.*, may also forward the message to its parent). On the other hand, the total energy $B_s(H)$ used by *any* structure H is at least $\sum_{e_i \in EMST} e_i^\beta/12$ [37]. Thus,

$$B_s(EMST) \geq \sum_{e_i \in EMST} e_i^\beta/12 = \omega_\beta(EMST)/12.$$

Then, if H is a low-weighted structure, we have

$$B_s(H) \leq 2 \sum_{e_i \in H} e_i^2 = O(n^{\beta-1}) \cdot \omega_\beta(EMST) \leq 12 \cdot O(n^{\beta-1}) \cdot B_s(EMST)$$

Consequently, we have the following lemma.

Lemma 2: The broadcast based on any low-weighted structure H consumes energy at most $O(n^{\beta-1})$ times of the minimum-energy broadcast. And the bound $O(n^{\beta-1})$ is tight.

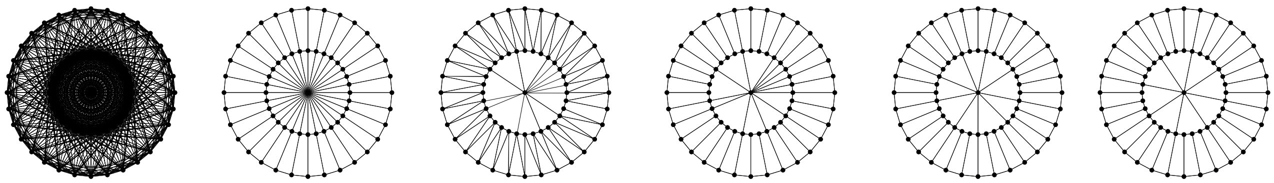
In summary, to enable energy efficient broadcasting, the locally constructed topology is also preferred to be *low-weighted*:

4. LOW WEIGHTED: the total link length of final topology is within a constant factor of that of $EMST$.

Recently, several localized algorithms [24], [26] have been proposed to construct low-weighted structures, which indeed approximate the energy efficiency of EMST as the network density increases. However, none of them is power efficient for unicast routing. In this paper we will present the first efficient distributed method to construct a planar, bounded degree spanner that is also low-weighted.

III. POWER-EFFICIENT UNICAST: SPANNER, PLANAR AND BOUNDED-DEGREE

The ultimate goal of this paper is to construct a unified topology that is power-efficient for both unicast and broadcast, in addition to be planar and have a constant bounded logical node degree. To achieve this ultimate goal, in this section, we first present a new method that can construct a power-efficient topology for unicast. We will prove that the constructed structure is a power-spanner, planar and has bounded node degree. Furthermore, it has an extra property: any two neighbors of each node are separated by at least a certain angle θ. Hereafter, we call it the Θ-*separation* property. As we will see later that this property further reduces the interference, especially when adopting directional antennas for transmission. This property also makes the proof much easier that the structure constructed in the next section is also power-efficient for broadcast.



(a) UDG (b) RNG, GG (c) BPS (d) $OrdYaoGG$ (e) $SYaoGG$ (f) $S\Theta GG$

Fig. 1. Several planar power spanners on the UDG shown in (a). Here $k = 9$ during constructing $SYaoGG$ and $S\Theta GG$.

One possible way to construct a degree-bounded planar power spanner is to apply the Yao structure on Gabriel graph, since GG is already planar and has a power stretch factor exactly 1. In [25], Li *et al.* showed that the final structure by directly applying the Yao structure on GG is a planar power spanner, called $YaoGG$, but its in-degree can be as large as $O(n)$, as in the example shown in Figure 1(b). In [34], Song *et. al* proposed two new methods to bound node degree by applying the ordered Yao structures on Gabriel graph. The structure $SYaoGG$ in [34] guarantees that there is at most one neighbor node in each of the k equal-sized cones. In this section, we will propose an improved algorithm to further reduce the medium contention by selecting less communication neighbors and separating neighbors wider.

Before we give the algorithm, we first define a concept called θ-Dominating Region.

Definition 1: θ-DOMINATING REGION: For each neighbor node v of a node u, the *θ-dominating region* of v is the 2θ-cone emanated from u, with the edge uv as its axis.

Using the concept of θ-dominating region instead of absolute cone partition in SYaoGG [34], our new method can further reduce the node degree bound by 1 and we are able to prove that any two neighbors of each node are guaranteed to be separated by at least an angle θ. We call this as Θ-separation property, which can further reduce interference especially while sending message through directional antennas. The final topology will be called $S\Theta GG$. Intuitively, the communication interference in $S\Theta GG$ will be smaller that the interference in $SYaoGG$, which is also verified later by simulations as shown in Figure 9(c) and (d).

The basic idea of our method is as follows. Since the Gabriel graph is planar and power-spanner, we will remove some links of GG to bound the nodal degree while not destroy the power-spanner property. The basic approach of bounding the nodal degree is to only keep some shortest link in the θ-Dominating region for every node. We process the nodes in a certain order. A node is marked WHITE if it is unprocessed and is marked BLACK if it is processed. Originally all nodes are marked WHITE. Initially, a node elects itself to start processing its neighbors if its ID[1] is smaller than all its unprocessed logical neighbors in the Gabriel graph. Assume that a node u is to be processed. We further assume that there are already some processed logical neighboring nodes, say $v_1, \cdots, v_t$, among its neighbors in GG. It keeps the link to the closest processed neighbor, say v_1, in GG, and removes all links to all neighbors in the *θ-dominating region* of v_1. In other words, the neighbor v_1 dominates all other neighbors in its θ-dominating region. It then repeats the above procedure until no processed logical neighbors in GG are left. Assume that node u also has some unprocessed logical neighbors, *i.e.*, marked WHITE. The node u then keeps the link to the closest unprocessed neighbor, say w, in GG if there is any, and then removes the links to all neighbors in the the *θ-dominating region* of w. It then repeats the above procedure until no unprocessed neighbors in GG are left. Node u then marks itself BLACK and then informs its logical neighbors in GG about its change of status. The algorithm terminates when all nodes are marked processed. The remaining links form the final structure, called $S\Theta GG$.

In our new algorithm, a data structure will be used: $N(u)$ is the set of neighbors of each node u in the final topology, which is initialized as the set of neighbor nodes in GG. We are now ready to present out algorithm, which constructs a degree-$(k-1)$ planar power spanner, as follows (see Algorithm 1).

It is easy to show that the final topology based on Yao graph, such as $SYaoGG$ [34], may vary as the choice of the direction of cones varies. Here, $S\Theta GG$ does not rely on the absolute cone partition by adopting the new concept of θ-dominating region. Hence, given the point set V, $S\Theta GG$ is unique. In addition, the average logical node degree, interference and transmission range of $S\Theta GG$ is expected to be smaller than $SYaoGG$

[1]It is not necessary to use ID here. We can also use some other mechanism to elect a certain node to perform the remaining procedures first. For example, we can use the RTS/CTS mechanism provided in the MAC layer to achieve this: the node that first successfully sent a RTS signal within its one-hop neighborhood will be elected. In this paper, we use ID just for the sake of easy presentation.

Algorithm 1 $S\Theta GG$: Power-Efficient Unicast Topology

1: First, each node self-constructs the Gabriel graph GG locally. The algorithm to construct GG locally is well-known, and a possible implementation may refer to [34]. Initially, all nodes mark themselves WHITE, *i.e.*, *unprocessed*.

2: Once a WHITE node u has the smallest ID among all its WHITE neighbors in $N(u)$, it uses the following strategy to select neighbors:

1. Node u first sorts all its BLACK neighbors (if available) in $N(u)$ in the distance-increasing order, then sorts all its WHITE neighbors (if available) in $N(u)$ similarly. The sorted results are then restored to $N(u)$, by first writing the sorted list of BLACK neighbors then appending the sorted list of WHITE neighbors.

2. Node u scans the sorted list $N(u)$ from left to right. In each step, it keeps the current pointed neighbor w in the list, while deletes every *conflicted* node v in the remainder of the list. Here a node v is conflicted with w means that node v is in the θ-dominating region of node w. Here $\theta = 2\pi/k$ ($k \geq 9$) is an adjustable parameter.

Node u then marks itself BLACK, *i.e. processed*, and notifies each deleted neighboring node v in $N(u)$ by a broadcasting message UPDATEN.

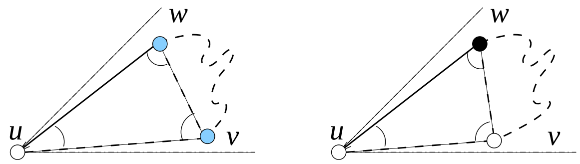
3: Once a node v receives the message UPDATEN from a neighbor u in $N(v)$, it checks whether itself is in the nodes set for deleting: if so, it deletes the sending node u from list $N(v)$, otherwise, marks u as BLACK in $N(v)$.

4: When all nodes are processed, all selected links $\{uv|v \in N(u), \forall v \in GG\}$ form the final network topology, denoted by $S\Theta GG$. Each node can shrink its transmission range as long as it sufficiently reaches its farthest neighbor in the final topology.

too. Furthermore, it is interesting to notice that the theoretical bound on the spanning ratio for $S\Theta GG$, that we can prove, is same as $SYaoGG$, as proved later in Theorem 4.

Lemma 3: Graph $S\Theta GG$ is connected if the underlying graph GG is connected. Furthermore, given any two nodes u and v, there exists a path $\{u, t_1, ..., t_r, v\}$ connecting them such that all edges have length less than $\sqrt{2}\|uv\|$.

Proof: We prove the connectivity by contradiction. Suppose a link uv is the shortest link in UDG whose connectivity is broken by Algorithm 1. W.l.o.g, assume the link uv is removed while processing node u, because of the existence of another node w.



(a) $\|uw\| < \|uv\|$ (b) $\|uw\| > \|uv\|$

Fig. 2. Two cases when uv is removed while processing u.

As shown in Figure 2, there are only two cases (ties are broken by ID) that the link uv can be removed by node u:

1. **Case a**: $\|uw\| < \|uv\|$. Notice that $\angle vuw \leq \theta < \pi/4$, hence $\|wv\| < \|uv\|$. In other words, both link wv and uw are smaller than link uv. Since there are no paths $u \leftrightsquigarrow v$ according to the assumption, either the path $u \leftrightsquigarrow w$ or $v \leftrightsquigarrow w$ is broken. That is to say, either the connectivity of wv or uw is broken. Thus, uv is not the shortest link whose connectivity is broken, it is a contradiction.

2. **Case b**: $\|uw\| > \|uv\|$. It happens only when node w is *processed* and node v is *unprocessed*. Similarly, $\angle vuw \leq \theta < \pi/4 < \angle uwv$ (otherwise $\angle uvw > \pi/2$ violates the Gabriel graph property), hence $\|wv\| < \|uv\|$. Since node w is a *processed* node and node u decides to keep link uw, the link uw will be kept in $S\Theta GG$. According to assumption that u and v are not connected in $S\Theta GG$, w and v are not connected either. That is to say, uv is not the shortest link whose connectivity is broken. It is a contradiction.

This finishes the proof of connectivity. Notice that the above proof implies that the shortest link uv in UDG is kept in the final topology. Clearly, the shortest link uv is in GG. Link uv cannot be removed in our algorithm due to the case illustrated by Figure 2 (a). Assume, for the sake of contradiction, that uv is removed due to the case (b) where $\|uw\| > \|uv\|$ and w is *processed* when processing u. Then $\|wv\| < \|uv\|$ is a contradiction to that uv is the shortest link in UDG.

We then show by induction that, given any link uv in UDG, there is a path connecting them using edges with length at most $\sqrt{2}\|uv\|$. Assume uv is removed when processing u, due to the existence of link uw. We build a path connecting u and v by concatenating $u \leftrightsquigarrow w$ and $w \leftrightsquigarrow v$, as shown in Figure 2. It is easy to see that the longest link of the path is less than $\sqrt{2}\|uv\|$, which occurs in case (b). In this case, the link uw must be kept because both endpoints are *processed*, and $\|uw\| < \sqrt{2}\|uv\|$. This finishes the proof. ■

The property that for any link uv, there is a path connecting them such that the links on the path have length at most $\sqrt{2}\|uv\|$ is crucial for our later proof that our Algorithm 2 builds a low-weighted bounded degree planar spanner.

Theorem 4: The structure $S\Theta GG$ has node degree at most $k-1$ and is planar power spanner with neighbors Θ-separated. Its power stretch factor is at most $\rho = \frac{\sqrt{2}^{\beta}}{1-(2\sqrt{2}\sin\frac{\pi}{k})^{\beta}}$, where $k \geq 9$ is an adjustable parameter.

Proof: The proof would be similar with the proof of $SYaoGG$ in [34]. The only difference is that, we used the concept of dominating cones instead of Yao graph. While the power stretch factor remains the same theoretically, the degree bound is reduced from k to $k-1$. Obviously, the links in $S\Theta GG$ are Θ-separated, in other words, the direction of any two neighbors of a node is Θ-separated. ∎

Figure 1 (e) and (f) show the difference of $SYaoGG$ and $S\Theta GG$. Compared with $SYaoGG$, $S\Theta GG$ is more evenly distributed and has a lower node degree.

IV. Unified Power-Efficient Topology: Degree-bounded Planar Spanner with Low Weight

To the best of our knowledge, so far, no localized topology control algorithm has achieved all the desirable properties summarized in Section II: *degree-bounded*, *planar*, *power spanner*, *low-weighted*. Those properties are not only interesting in terms of computational geometry, but also have important applications in wireless ad hoc networks, as shown in section II: enable energy efficient unicast and broadcast routings in same structure. Recall that, spanner property ensures that an energy efficient path is always kept for any pair of nodes, hence it is a necessary condition to support energy efficient unicast. While low-weighted structure is optimal for broadcast among any connected structures built in localized manner. Unfortunately, all the known spanners, including Yao [42], GG [11] and the recent developed degree-bounded planar spanners BPS [38], $SYaoGG$, $OrdYaoGG$ [34] and $S\Theta GG$, are not low-weighted. As illustrated in Figure 1, all of them will keep at least $\frac{n-1}{2}$ links between the two circles, while EMST (in Figure 4(b)) will keep only one link between them. Hence the weight of any of them is at least $O(n) \cdot w(EMST)$.

Worth to clarify that, in this section, we are interested in finding a subgraph to enable efficient broadcast routings, *even* based on the simple-flooding method. We do *not* aim to substitute known broadcasting protocols. In fact, the methods used in those localized broadcasting protocols [41], [6] can be applied on the low-weighted structures to conserve more energy. The main contribution of low-weighted structure is that it bounds the worst case performance for broadcasting.

Several known localized algorithms are given in [24], [26] to generate low-weighted graphs. In their algorithms, given a certain structure G, for any two links uv and xy of a graph G, they remove xy if xy is the longest link among quadrilateral $uvxy$. They proved that the final structures are low-weighted if G is RNG' [24] or LMST$_2$ [26]. Obviously, they are not spanners. In fact, their techniques can *not* be applied to spanner graphs to bound the weight without losing the spanner property. Figure 3 illustrates an example by applying their algorithms to $S\Theta GG$. The node ID of v_i is i, $\angle v_1v_3v_4 < \theta$ and

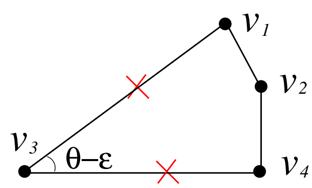
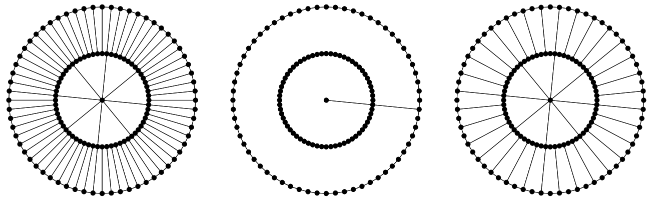


Fig. 3. The graph could be disconnected if applying the previous method to build low-weighted structure on $S\Theta GG$.

$\|v_1v_3\| > \|v_3v_4\| > \max(\|v_1v_2\|, \|v_2v_4\|)$. While constructing $S\Theta GG$, first node v_1 selects v_1v_2 and v_1v_3 as its incident logical links and node v_2 selects v_2v_1 and v_2v_4, then node v_3 selects v_3v_1 and deletes v_3v_4. Hence $v_3v_4 \notin S\Theta GG$. If applying the rule described in [24], [26], the link v_1v_3 will also be deleted because $\|v_1v_3\| > \max(\|v_1v_2\|, \|v_2v_4\|, \|v_3v_4\|)$. Then the graph will be disconnected. Then we can conclude that simple extension of methods in [26] on top of $S\Theta GG$ does not even guarantee the connectivity, nor to say *power-spanner* property.

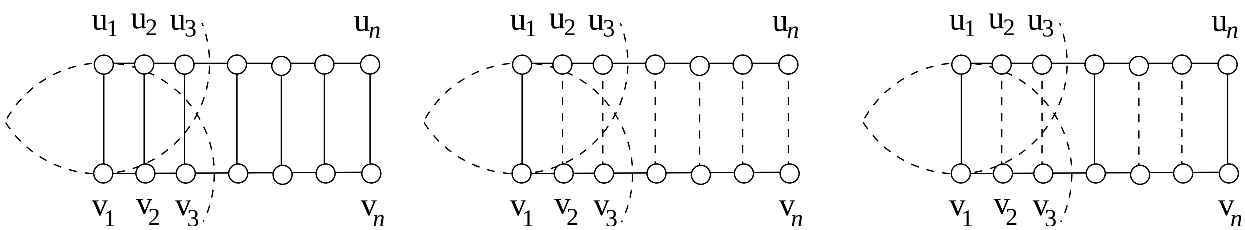
Indeed, the spanner property and low-weight property are *not* easy to be achieved at same time. Intuitively, the spanner property requires to keep more links, while the low-weight property requires to keep less links from original graph. In the following, we will describe a novel algorithm to build a low-weighted structure from $S\Theta GG$, while keeping enough links to guarantee the power efficiency. Figure 4 illustrates the difference of $LS\Theta GG$ from $S\Theta GG$ and $LMST_2$.



(a) $S\Theta GG$ (b) $LMST_2$ (c) $LS\Theta GG$

Fig. 4. The difference between $LS\Theta GG$, $S\Theta GG$ and $LMST_2$.

Algorithm 2 presents our new method that constructs a bounded degree planar power-spanner that is also low-weighted. Although our algorithm produces only power-spanner here, it can be extended to produce also the length-spanner if it is needed. To get a length-spanner, we construct the structure $LDel^2$ (defined in [39]) instead of the Gabriel graph used in our algorithm. It was proved in [39] that $LDel^2$ is a planar, length-spanner, and can be constructed using only $O(n)$ messages. The basic idea of our new method is as follows. Since the graph $S\Theta GG$ is already planar, power-spanner, and has bounded-degree, we will remove some of its edges to guarantee that the resulting topology is low-weighted while not destroying the power-spanner property. Notice that removing edges will not break the planar property and the bounded-degree property. In all previous methods presented in the literature, a node x decides to remove or keep links that are incident on x, *i.e.*, it only cares about the incident edges. While, in the method presented here, a node x will decide whether to keep or remove links for not only incident links, but also the links that are incident on one of its neighbors. To guarantee a low-weight property the methods presented in [24], [26] remove some links from a certain structure such that the remaining links satisfy the *isolation property*: for each remaining link xy, the disk centered at the midpoint of xy using a radius proportional to $\|xy\|$ does not intersect with any other remaining links. They achieved this property by removing a link xy if there is another link uv such that xy is the longest link in the quadrilateral $uvyx$. However, this simple heuristic cannot guarantee the spanner property. Consider a link xy in some shortest path from s to t. See Figure 6 for an illustration. Link xy will be removed due to the existence of link uv. Link uv could also later be removed due to the existence of another link u_1v_1, which could also be removed due to the existence of another link u_2v_2, and so on. See Figure 5 (b) for an illustration of the situation where a sequence of links will be removed: all links u_iv_i, for $i \geq 2$ will be removed. Consequently, the shortest path connecting nodes u_n and v_n could be arbitrarily long in the resulting graph.



(a) original graph $S\Theta GG$ (b) graph resulted using [24] (c) graph based on our method

Fig. 5. A sequence of links are recursively removed. Here solid links represent the links from the original graph and the dashed links represent the links that are removed by a topology control algorithm. Here we assume that $\|u_iv_i\| = R$ and the ID of link u_iv_i is less than the ID of link $u_{i+1}v_{i+1}$.

Thus, instead of blindly removing all such links xy whenever it is the longest link in a quadrilateral $uvyx$, we will keep such a link when the links in its certain neighborhood have been removed. To do so, among all links from a graph, such as $S\Theta GG$, that is planar, bounded-degree, power-spanner, we *implicitly* define an independent set of links. A link is in this independent set, which will be kept at last, if it has the smallest ID among unselected links from its neighborhood. Specifically, we implicitly define a virtual graph G' over all links in $S\Theta GG$: the vertex set of G' is the set of all links in $S\Theta GG$ and two links xy and uv of $S\Theta GG$ are connected in G' if one end-point of uv is in the transmission range of one end-point of link xy. For example, the links u_1v_1 and u_3v_3 are not independent in network topology illustrated by Figure 5 (a); while the links u_1v_1 and u_nv_n are independent. Notice that links u_1v_1 and u_1u_2 are independent since they do not form a four vertices convex hull. Notice that in our method presented later, we did not explicitly define such graph G', nor compute the maximal independent set of such graph G' explicitly. We will prove that the selected independent set of links in $S\Theta GG$ indeed is low-weighted and still preseves the power-spanner property, although with a larger power spanning ratio. Our method will keep link u_1v_1 since it has the smallest ID among all links that are not independent. When link u_1v_1 is kept, all links that are not independent (here are u_2v_2 and u_3v_3) will be removed. Then link u_4v_4 will be kept. The above procedure will be repeated until all links are processed. The final structure resulted from our method is illustrated by Figure 5 (c).

Algorithm 2 Construct $LS\Theta GG$: Planar Spanner with Bounded Degree and Low Weight

1: All nodes together construct the graph $S\Theta GG$ in a localized manner, as described in Algorithm 1. Then, each node marks its incident edges in $S\Theta GG$ *unprocessed*.

2: Each node u locally broadcasts its incident edges in $S\Theta GG$ to its one-hop neighbors and listens to its neighbors. Then, each node x can learn the existence of the set of 2-hop links $E_2(x)$, which is defined as follows: $E_2(x) = \{uv \in S\Theta GG \mid u$ or $v \in N_{UDG}(x)\}$. In other words, $E_2(x)$ represents the set of edges in $S\Theta GG$ with at least one endpoint in the transmission range of node x.

3: Once a node x learns that its *unprocessed* incident edge xy has the smallest ID among all *unprocessed* links in $E_2(x)$, it will delete edge xy if there exists an edge $uv \in E_2(x)$ (here both u and v are different from x and y), such that $\|xy\| > \max(\|uv\|, 3\|ux\|, 3\|vy\|)$; otherwise it simply marks edge xy *processed*. Here assume that $uvyx$ is the convex hull of u, v, x and y. Then the link status is broadcasted to all neighbors through a message UPDATESTATUS(XY).

4: Once a node u receives a message UPDATESTATUS(XY), it records the status of link xy at $E_2(u)$.

5: Each node repeats the above two steps until all edges have been *processed*. Let $LS\Theta GG$ be the final structure formed by all remaining edges in $S\Theta GG$.

Obviously, the construction is consistent for two endpoints of each edge: if an edge uv is kept by node u, then it is also kept by node v. Worth to mention that, the number 3 in criterion $\|xy\| > \max(\|uv\|, 3\|ux\|, 3\|vy\|)$ is carefully selected, as we will see later that .

Theorem 5: The structure $LS\Theta GG$ is a degree-bounded planar spanner. It has a constant power spanning ratio $2\rho + 1$, where ρ is the power spanning ratio of $S\Theta GG$. The node degree is bounded by $k - 1$ where $k \geq 9$ is a customizable parameter in $S\Theta GG$.

Proof: The degree-bounded and planar properties are obviously derived from the $S\Theta GG$ graph, since we do not add any links in Algorithm 2.

To prove the spanner property, we only need to show that the two endpoints of any deleted link $xy \in S\Theta GG$ is still connected in $LS\Theta GG$ with a constant spanning ratio path. We will prove it by induction on the length of deleted links from $S\Theta GG$.

Assume xy is the shortest link of $S\Theta GG$ which is deleted by Algorithm 2 because of

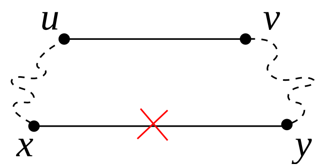


Fig. 6. The path between x and y is at most $(2\rho+1)\|xy\|$ in $LS\Theta GG$ if $xy \in S\Theta GG$.

the existence of link uv with smaller length. Obviously, path $x \leftrightsquigarrow y$ can be constructed through the concatenation of path $x \leftrightsquigarrow u$, link uv and path $v \leftrightsquigarrow y$, as shown in Figure 6. Since $\|xy\| > \max(\|ux\|, \|vy\|)$ and link xy is the shortest among deleted links in Algorithm 2, we have $p(x \leftrightsquigarrow u) < \rho\|ux\|^\beta$ and $p(v \leftrightsquigarrow y) < \rho\|vy\|^\beta$. Hence,

$$\begin{aligned} p(x \leftrightsquigarrow y) &< \|uv\|^\beta + \rho\|ux\|^\beta + \rho\|vy\|^\beta \\ &< (2\rho+1)\|xy\|^\beta. \end{aligned}$$

Suppose all the i-th ($i \le t-1$) deleted shortest links of $S\Theta GG$ have a path connecting their endpoints with spanning ratio $2\rho+1$. For the t-th deleted shortest link $xy \in S\Theta GG$, according to Algorithm 2, it must have been deleted because of the existence of a link uv: such that $\|xy\| > \max(\|uv\|, 3\|ux\|, 3\|vy\|)$ in a convex hull $uvyx$. Now, we have $p(x \leftrightsquigarrow u) < (2\rho+1)\|ux\|^\beta$ and $p(v \leftrightsquigarrow y) < (2\rho+1)\|vy\|^\beta$. Thus,

$$\begin{aligned} p(x \leftrightsquigarrow y) &= \|uv\|^\beta + p(u \leftrightsquigarrow x) + p(v \leftrightsquigarrow y) \\ &< \|uv\|^\beta + (2\rho+1)\|ux\|^\beta + (2\rho+1)\|vy\|^\beta \\ &< \|xy\|^\beta + (2\rho+1)(\|xy\|/3)^\beta + (2\rho+1)(\|xy\|/3)^\beta \\ &\le (2\rho+1)\|xy\|^\beta \end{aligned}$$

Thus, $LS\Theta GG$ has a power spanning ratio $\le 2\rho+1$. ■

We then show that graph $LS\Theta GG$ is low-weighted. To study the total weight of this structure, inspired by the method proposed in [24], we will show that the edges in $LS\Theta GG$ satisfy the *isolation property* [9].

Theorem 6: The structure $LS\Theta GG$ is low-weighted.

See the appendix for the proof. We continue to analyze the communication cost of Algorithm 1 and 2. First, clearly, building GG in Algorithm 1 can be done using only n messages: each message contains the ID and geometry position of a node. Second, to build

$S\Theta GG$, initially, the number of edges, say p, in Gabriel Graph is $p \in [n, 3n-6]$ since it is a planar graph. Remember that we will remove some edges from GG to bound the logical node degree. Clearly, there are at most $2n$ such removed edges since we keep at least $n-1$ edges from the connectivity of the final structure. Thus the total number of messages, say q, used to inform the deleted edges from GG is at most $q \in [0, 2n]$. Notice that $p-q$ is the edges left in the final structure, which is at least $n-1$ and at most $3n-6$. Thirdly, in the marking process described in Algorithm 2, the communication cost of broadcasting its incident edges (or its neighbors) and updating link status are both $2(p-q)$. Therefore the total communication cost is $n+4p-3q \in [5n, 13n]$. Then the following theorem directly follows.

Theorem 7: Assuming that both the ID and the geometry position can be represented by $\log n$ bits each, the total number of messages during constructing the structure $LS\Theta GG$ is in the range of $[5n, 13n]$, where each message has at most $O(\log n)$ bits.

Compared with previous known low-weighted structures [24], [26], $LS\Theta GG$ not only achieves more desirable properties, but also costs much less messages during construction. To construct $LS\Theta GG$, we only need to collect the information $E_2(x)$ which costs at most $6n$ messages. Our algorithm can be generally applied to any known degree-bounded planar spanner to make it low-weighted while keeping all its previous properties, except increasing the spanning ratio from ρ to $2\rho+1$ theoretically.

V. Expected Interference in Random Networks

This section is devoted to study the average physical node degree of our structure when the wireless nodes are distributed according to a certain distribution. For average performance analysis, we consider a set of wireless nodes distributed in a two-dimensional unit square region. The nodes are distributed according to either the uniform random point process or homogeneous Poisson process. A point set process is said to be a *uniform random point process*, denoted by $\mathcal{X}_n$, in a region Ω if it consists of n independent points each of which is uniformly and randomly distributed over Ω. The standard probabilistic model of *homogeneous Poisson process* is characterized by the property that the number of nodes in a region is a random variable depending only on the area of the region, *i.e.*, (1) The probability that there are exactly k nodes appearing in any region Ψ of area A is

$\frac{(\lambda A)^k}{k!} \cdot e^{-\lambda A}$; (2) For any region Ψ, the conditional distribution of nodes in Ψ given that exactly k nodes in the region is *joint uniform*.

Definition 2: Given a structure H, the adjusted transmission range $r_H(u)$ is defined as $\max_{uv \in H} \|uv\|$, *i.e.*, the longest edge of H incident on u. The physical node degree u in H is defined as the number of nodes inside the disk $disk(u, r_H(u))$. The node interference, denoted by $I_H(u)$, caused by a node u in a structure H is simply the physical node degree of u. The maximum node interference of a structure H is defined as $\max_u I_H(u)$. The average node interference of a structure H is defined as $\sum_u I_H(u)/n$.

Theorem 8: For a set of nodes produced by a Poisson point process with density n, the expected maximum node interferences of EMST, GG, RNG and Yao are at least $\Theta(\log n)$.

Proof: Let d_n be the longest edge of the EMST of n points placed independently in 2-dimensions according to standard Poisson distribution with density n. In [29], they showed that

$$\lim_{n \to \infty} P_r(n\pi d_n^2 - \log n \leq \alpha) = e^{-e^{-\alpha}}.$$

Notice that the probability $P_r(n\pi d_n^2 - \log n \leq \log n)$ will be sufficiently close to 1 when n goes to infinity, while the probability $P_r(n\pi d_n^2 - \log n \leq -\log\log n)$ will be sufficiently close to 0 when n goes to infinity. That is to say, with high probability, $n\pi d_n^2$ is in the range of $[\log n - \log\log n, 2\log n]$.

Given a region with area A, let $m(A)$ denote the number of nodes inside this region by a Poisson point process with density δ. According to the definition of Poisson distribution, $P_r(m(A) = k) = \frac{e^{-\delta A}(\delta A)^k}{k!}$. Thus, the expected number of nodes lying inside a region with area A is $E(m(A)) = \sum k \cdot P_r(m(A) = k) = \sum_{k=1}^{\infty} \frac{e^{-\delta A}(\delta A)^k}{k!} k = \delta A \sum_{k=1}^{\infty} \frac{e^{-\delta A}(\delta A)^{k-1}}{(k-1)!} = \delta A$. For a Poisson process with density n, let uv be the longest edge of the Euclidean minimum spanning tree, and $d_n = \|uv\|$. Then, the expectation of the number of nodes that fall inside $disk(u, d_n)$ is $E(m(\pi d_n^2)) = n\pi d_n^2$, which is larger than $\log n$ almost surely when n goes to infinity. That is to say, the expected maximum interference of EMST is $\Theta(\log n)$ for a set of nodes produced according to a Poisson point process. Consequently, the expected maximum node interference of any structure containing EMST is at least $\Omega(\log n)$. Thus, the expected maximum node interference of structure GG, RNG and Yao structures are also at least $\Omega(\log n)$. A similar analysis can show that all commonly used structures for topology control in wireless ad hoc networks generally have a large maximum node interference even for nodes deployed with uniform random distribution. ■

Our following analysis will show that the average interference of all nodes of these structures is small for a randomly deployed network. Notice that all our following results also hold for nodes deployed with uniform random distribution.

Theorem 9: For a set of nodes produced by a Poisson point process with density n, the expected average node interferences of EMST are bounded from above by a constant.

Proof: Consider a set V of wireless nodes produced by Poisson point process. Given a structure G, let $I_G(u_i)$ be the node interference caused by (or at) a node u_i, *i.e.*, the number of nodes inside the transmission region of node u_i. Here the transmission region of node u_i is a disk centered at u_i with radius $r_i = \max_{u_i v \in G} \|u_i v\|$. Hence, the expected average node interference is

$$E(\frac{\sum_{i=1}^{n} I_G(u_i)}{n}) = \frac{1}{n} E(\sum_{i=1}^{n} I_G(u_i)) = \frac{1}{n} \sum_{i=1}^{n} E(I_G(u_i))$$
$$= \frac{1}{n} \sum_{i=1}^{n} E(m(\pi r_i^2)) = \frac{1}{n} \sum_{i=1}^{n} (n \pi r_i^2)$$
$$= \sum_{i=1}^{n} (\pi r_i^2) \leq 2 \sum_{e_i \in G} (\pi e_i^2).$$

The last inequality follows from the fact that r_i is the length of some edge in G and each edge in G can be used by at most two nodes to define its radius r_i.

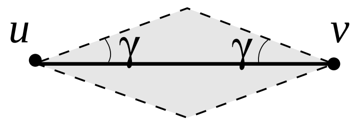
Let e_i, $1 \leq i \leq n-1$ be the length of all edges of the EMST of n points inside a unit disk. It was shown in [37] that $\sum_{e_i \in EMST} e_i^2 \leq 12$. Thus, the expected average node interference of the structure EMST is

$$E(\frac{\sum_{i=1}^{n} I_{EMST}(u_i)}{n}) \leq 2 \sum_{e_i \in EMST} (\pi e_i^2) \leq 24\pi.$$

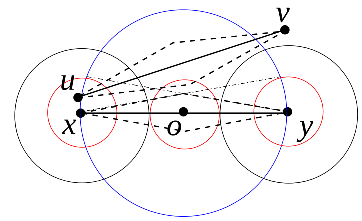
This finishes our proof. ■

Theorem 10: The expected average node interferences of $LS\Theta GG$ are bounded from above by a constant.

Proof: We prove it by showing that in $LS\Theta GG$, all the diamonds $D(uv, \gamma)$ subtended from each link segment $uv \in LS\Theta GG$ do not overlap with each other, where $\sin 2\gamma = \frac{1}{3}$.



(a) The diamond subtended from link uv



(b)Any pair of diamonds do not overlap

Fig. 7. The proof illustration of expected average node interference

Here, the diamond $D(uv, \gamma)$ is defined as the rhombus subtended from a line segment uv, with sides of length $\|uv\|/(2\cos\gamma)$, where $0 \leq \gamma \leq \pi/3$ is a parameter. See Figure 7 (a) for an illustration.

Figure 7 (b) illustrates the basic idea of the proof. For any two segments uv and xy, we can show that either the angle between them is at least 2γ (implies that two diamonds $D(uv, \gamma)$ and $D(xy, \gamma)$ do not overlap), or the distance between them is far enough to separate these two diamonds. The detail of the proof is omitted here due to space limit and it is not difficult to verify. It is easy show that the total area of these diamonds is $\frac{\tan\gamma}{2}\sum_{e_i \in LS\Theta GG} e_i^2 \simeq 0.084\sum_{e_i \in LS\Theta GG} e_i^2$. Then we can show that $\sum_{e_i \in LS\Theta GG} e_i^2 \leq 12\pi$. Thus, the expected average node interference is at most $2\sum_{e_i \in LS\Theta GG} e_i^2 \leq 24\pi$. ■

VI. Performance on Random Networks

In this section we evaluate the performance of our new energy efficient unicast and broadcast topology $S\Theta GG$ by conducting simulations. In our experiments, we randomly generate a set V of n wireless nodes and $UDG(V)$, then test the connectivity of $UDG(V)$. If it is connected, we construct different localized topologies on $UDG(V)$, including our new topology $S\Theta GG$ and some well-known planar topologies GG [11], RNG [35], and $SYaoGG$ [34]. Then we measure the sparseness, the power efficiency and the interference of these topologies.

In the experimental results presented here, we generate n random wireless nodes in a 20×20 unit squares; the parameter k is set to 9 when we construct $SYaoGG$ and $S\Theta GG$; the transmission range is set to 8 unit. Typically, a unit represents about 50 meters here. We test the power efficiency, and node degree of these planar structures by varying node number from 30 to 300, where 100 vertex sets are generated for each case to smooth the possible peak effects caused by some exception examples. The average and the maximum are computed over all these 100 vertex sets.

A. Power Efficiency for Unicast

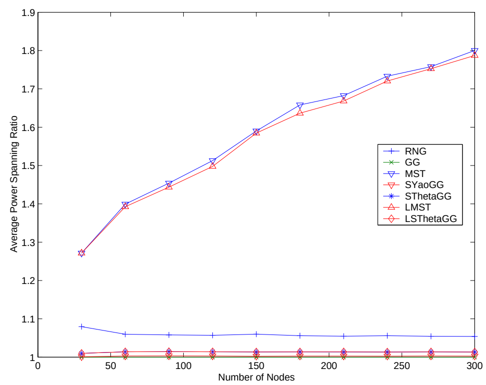
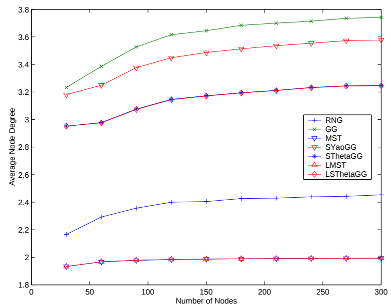


Fig. 8. Average power spanning ratio of different topologies.

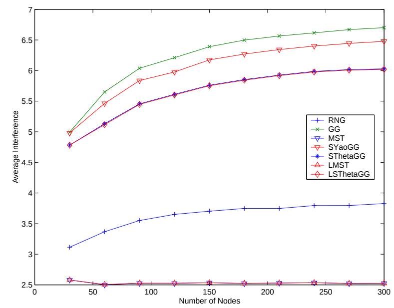
The most important design metric of wireless network topology is perhaps the power efficiency, as it directly affects both the node and the network lifetime. First, we test power stretch factors of all structures. In our simulations we set power attenuation constant $\beta = 2$. Figure 8 summarizes our experimental results of power stretch factors of all these topologies. It shows all power spanners (GG, $SYaoGG$, $S\Theta GG$, $LS\Theta GG$) indeed have small power spanning ratio in practice: less than 1.021, while RNG, $LMST_2$, $EMST$ are less power efficient as proved. Hence, for unicast application, we only need compare the performance among power spanners. The average power stretch factors of $LS\Theta GG$ are at the same level of those of GG though they are sparser and low-weighted.

B. Logical Node Degree

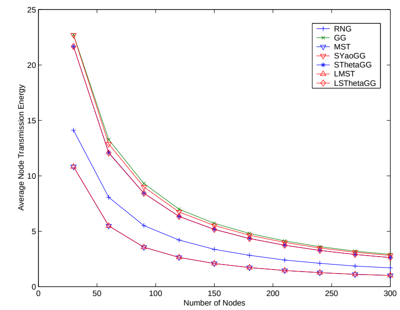
In unicast routings, each node is preferred to have bounded number of communication neighbors. Otherwise a node with large degree has to communicate with many nodes directly. This increases the interference and the overhead at this node. The average and maximum logic node degrees of each topology are shown in Figure 9 (a) and (b). It shows that $S\Theta GG$ and $LS\Theta GG$ have less number of edges (average node degrees) than $SYaoGG$ and GG. In other words, our new structure is sparser.



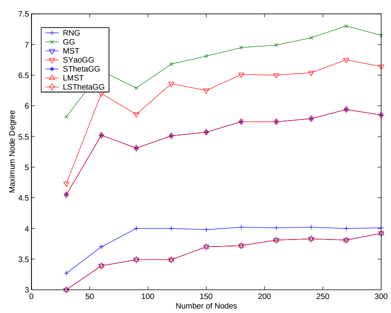
(a) Average logical degree



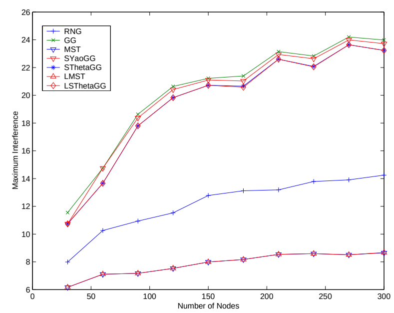
(c) Average physical degree



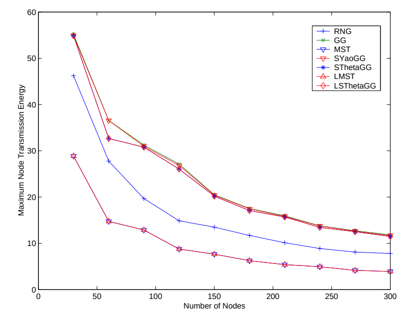
(e) Average node power



(b) Maximum logical degree



(d) Maximum physical degree



(f) Maximum node power

Fig. 9. The average and maximum performances of various structures.

C. *Physical Node Degree*

Beside the logical node degrees of all these structures, we are also interested in another kind of node degrees, called *physical node degrees* (or called node interference) and defined as follows. For each node u, it has a longest link, say uv, in a constructed structure. Then the node interference of u is defined as all nodes w such that $\|uw\| \leq \|uv\|$. This is the total number of nodes that could cause direct interference with u. The average and maximum node interference of each topology are shown in Figure 9 (c) and (d). They are higher than the logical node degrees as expected, however they follow the same pattern of curves. Moreover, the possible maximum interference increases slightly when the number of wireless nodes grows. As predicted in section III, both average and maximum node interference of $S\Theta GG$ are lower than $SYaoGG$. The average node interference of $LS\Theta GG$ is indeed bounded, which is around 6 in our simulations.

D. Power Assignment for Broadcast

After forming the sparse structures, for broadcast, each node can shrink its transmission energy as long as it is enough to cover the longest adjacent neighbor in the structure. By this way, we define the node transmission power for each node u in a constructed structure as follows. If u has a longest link, say uv, in the structure, then the node transmission energy of u is $\|uv\|^{\beta}$. Recall the discussion in section II-B, once a structure is constructed, the broadcast is simple flooding: every node will forward the received broadcast message once to all logical neighbors in the structure.

The average and the maximum node transmission energy of each topology are shown in Figure 9 (e) and (f), which decrease as the network density increases as expected. More importantly, the power assignment based on $LS\Theta GG$ is smaller than RNG, which has been widely used for broadcasting previously. Our theoretical results are corroborated in the simulations: low-weighted structure is indeed close to optimal for broadcast among all locally constructed structures.

Moreover, simulation results in all charts also show that the performances of our new topologies $LS\Theta GG$ are stable when the number of nodes changes.

VII. Conclusion

Energy conservation is critical to the network performance in wireless ad hoc networks. Topology control has drawn significant research interests from different approaches for energy conservation in wireless ad hoc networks. In this paper, we proposed an efficient algorithm in which all wireless nodes maintain an network topology, called $LS\Theta GG$, which is the first known single structure to support both energy efficient unicast and broadcast. We gave distributed method to construct it with $5n$ to $13n$ messages. We proved that, for unicast, it has following attractive properties: power spanner, bounded node degree, planar, and low average interference. Furthermore, the total energy of broadcast based on this structure is also within a constant factor of the power consumption of broadcast based on any locally constructed topology. Previous known localized topology control algorithms can only achieve part of those nice properties, especially, none of them can support both efficient unicast and broadcast simultaneously. As a future work, we would like to design energy efficient structures by considering also the power consumed by the receiving nodes. To design a structure that has the smallest possible physical degree while achieving a certain property (such as efficient unicast, efficient broadcast or planar) is also very interesting.

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Appendix

Das *et al.* [9] proved that if a set of line segments E satisfies the isolation property, then $\omega(E) = O(1)\cdot\omega(SMT)$. Here SMT is the Steiner minimum tree over the end points of E, and total edge weight of SMT is no more than that of the minimum spanning tree. The isolation property is defined as follows. Let $c > 0$ be a constant and E be a set of edges in d-dimensional space, and let $e \in E$ be an edge of length l. If it is possible to place a *protecting disk* B of radius $c \cdot l$ with center on e and B does not intersect with any other edge, then edge e is said to be *isolated* [9]. If all the edges in E are isolated, then E is said to satisfy the *isolation property*. We define the *protecting disk* of a segment uv as $disk(o, \frac{\sqrt{35}}{36}\|uv\|)$, where o is the midpoint of segment uv. Obviously, we need all such disks do not intersect any edge except the one that defines it.

Theorem 6: The structure $LS\Theta GG$ is low-weighted.

Proof: We will prove this by showing that all edges E in $LS\Theta GG$ satisfy the isolation property. For the sake of contradiction, assume that E does not satisfy the isolation property.

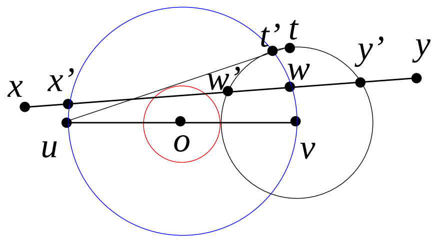
Assume there is one edge uv that is not isolated. Thus, there is an edge, say xy, that intersects the protecting disk of uv. Figure 10 illustrates the hypothetical situation: a link xy intersects the protecting disk of link uv, *i.e.*, $disk(o, \frac{\sqrt{35}}{36}\|uv\|)$. First notice that, both x and y can not locate inside $disk(o, \frac{1}{2}\|uv\|)$, otherwise the property of Gabriel graph is violated.

We further divide the hypothetical situation into two cases:

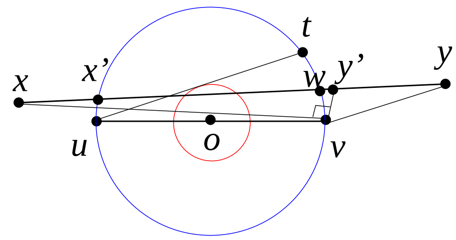
Case 1: $\|xy\| < \|uv\|$.

We will show that the link uv itself must have been removed by our algorithm, by proving that both $\|ux\|$ and $\|vy\|$ are no more than $\frac{1}{3}\|uv\|$ in the hypothetical situation. To prove this by inducing contradiction, w.l.o.g., we assume that $\|vy\| > \frac{1}{3}\|uv\|$.

Figure 10(a) illustrates our proof that follows. The link xy intersects the $disk(o, \frac{1}{2}\|uv\|)$ with two points x' and w, and intersects the right half of $disk(v, \frac{1}{3}\|uv\|)$ with the point y'. Let t be a point on the top half of $disk(v, \frac{1}{3}\|uv\|)$ such that $\|ut\| = \|uv\|$. The segment ut intersects the $disk(o, \frac{1}{2}\|uv\|)$ with



(a)$\|xy\| < \|uv\|$



(b)$\|xy\| > \|uv\|$

Fig. 10. Two hypothetical cases that an edge uv is not isolated

point t'. It is easy to verify that ut is the tangent line of protecting $disk(o, \frac{\sqrt{35}}{36}\|uv\|)$.

From the assumption $\|vy\| > \frac{1}{3}\|uv\|$, node y is out of the $disk(v, \frac{1}{3}\|uv\|)$. Hence, $\|xy\| > \|x'y'\|$. We continue to induce contradiction that $\|xy\| > \|uv\|$ by showing $\|x'y'\| > \|ut\| = \|uv\|$.

1. Obviously, $\|x'w\| > \|ut'\|$, because the chord $x'w$ of $disk(o, \frac{1}{2}\|uv\|)$ is closer the center o than the chord ut' (because $x'w$ intersects the protecting disk while ut is the tangent line).

2. Similarly, $\|wy'\| > \|tt'\|$, because $\|ww'\| > 2\|tt'\|$ (In $disk(v, \frac{1}{3}\|uv\|)$, the chord ww' is closer to center v than line tt', and segment tt' is half of the chord overlapping tt' since vt' is perpendicular to ut' in $disk(o, \frac{1}{2}\|uv\|)$.) and $\|wy'\| > \frac{1}{2}\|ww'\|$ (In $disk(v, \frac{1}{3}\|uv\|)$, $\angle x'wv > \angle uwv = \frac{\pi}{2}$, hence $\|wy'\|$ is more than half of the chord ww'.).

Consequently, $\|xy\| > \|x'y'\| = \|x'w\| + \|wy'\| > \|ut'\| + \|tt'\| = \|ut\| = \|uv\|$, hence we get the contradiction. In other words, uv should have been deleted if $\|uv\| > \|xy\|$ and xy intersects the protecting disk of uv. The hypothetical case is fake.

Case 2: $\|xy\| > \|uv\|$.

We will show that xy will be deleted by Algorithm 2 by showing $\max(\|ux\|, \|vy\|) < \frac{1}{3}\|xy\|$ if xy intersects the protecting $disk(o, \frac{\sqrt{35}}{36}\|uv\|)$ of link uv. We prove it by inducing contradiction. W.l.o.g., assume that $\|vy\| > \frac{1}{3}\|xy\|$.

Figure 10(b) illustrates our proof that follows. Here ut is a tangent line of the protecting disk, the link xy intersects the $disk(o, \frac{1}{2}\|uv\|)$ with two points x' and w. The segment $y'v$ is perpendicular to xv. Here point y' is on line xy.

Obviously, $\angle vxy < \angle vx'y$. And $\angle vx'y < \angle vut$ because the arc $\widehat{vw}$ is smaller than the arc $\widehat{vt}$. We have, $\|vy'\| = \|xy'\|\sin(\angle vxy) < \|xy'\|\sin(\angle vut) = \frac{\sqrt{35}}{18}\|xy'\| < \frac{1}{3}\|xy\|$. On the other hand, $\|vw\| < \|vt\| < \frac{1}{3}\|uv\| < \frac{1}{3}\|xy\|$. Hence, node y can not be on the left side of y', instead only possible on the right side since $\|vy\| > \frac{1}{3}\|xy\|$. Then, we have $\angle xvy > \frac{\pi}{2}$, *i.e.*, link xy cannot be in GG. Contradiction is induced.

Consequently, xy should have been deleted if $\|uv\| < \|xy\|$ and xy intersects the protecting disk of uv. The hypothetical case is also fake.

In summary, each link $uv \in LS\Theta GG$ satisfies the isolation property, that is to say, $LS\Theta GG$ is low-weighted. This finishes the proof. ■