# Localized Topology Control for Heterogeneous Wireless Sensor Networks

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The paper studies topology control in heterogeneous wireless sensor networks, where different wireless sensors may have different maximum transmission ranges and two nodes can communicate directly with each other if and only if they are within the maximum transmission range of each other. We present several localized topology control strategies in which every wireless sensor maintains logical communication links to only a selected small subset of its physical neighbors using information of sensors within its local neighborhood in heterogeneous network environment. We prove that the global logical network topologies formed by these locally selected links are sparse and/or power efficient and our methods are communication efficient. Here a structure is power efficient if the total power consumption of the least cost path connecting any two nodes in it is no more than a small constant factor of that in the original heterogeneous communication network. By utilizing the wireless broadcast channel capability, and assuming that a message sent by a sensor node will be received by all sensors within its transmission region with at most a constant number of transmissions, we prove that all our methods use at most $O(n)$ total messages, where each message has $O(\log n)$ bits. We also conduct extensive simulations to study the practical performances of our methods.

Categories and Subject Descriptors: C.2.1 [**Network Architecture and Design**]: Wireless communication, Network topology; G.2.2 [**Graph Theory**]: Network problems, Graph algorithms

General Terms: Algorithms, Design, Theory

Additional Key Words and Phrases: Graph theory, wireless sensor networks, topology control, heterogeneous networks, power consumption, degree-bounded structure.

## 1. INTRODUCTION

In wireless *ad hoc* networks, *e.g.*, the wireless sensor networks, an important requirement is that the network should be self-organized, *i.e.*, transmission ranges and data paths are dynamically restructured with changing network conditions, especially the connectivity. In wireless sensor networks, the network has to determine for each sensor which communication links to its physical neighboring sensors it has to maintain such that the global logical network topology satisfies some

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properties that are important for the long time network performances. Localized wireless network topology control scheme is to let each wireless node locally adjust its transmission power and select which physical neighbors to communicate according to a certain strategy, while maintaining a structure that can support energy efficient routing and can improve the overall network performances. Hence it can efficiently conserve the transmission energy from soft aspects with low cost.

In the past several years, topology control algorithms have drawn a significant amount of research interests. Centralized algorithms can achieve optimality or some approximations, which are more applicable to static network conditions due to the lack of adaptability to dynamic changes. In contrast, distributed algorithms are more suitable for dynamic wireless sensor networks since the environment is inherently dynamic and they are adaptive to topology changes at the cost of possible less optimality. Furthermore, these localized topology control algorithms, run by each individual node, only attempt to selectively choose some communication neighbors of this node. The primary distributed topology control algorithms for ad hoc networks and sensor networks aim to maintain network connectivity, optimize network throughput with power-efficient routing, conserve energy and increase the fault tolerance.

Most prior art [Hu 1993; Li et al. 2001; Li et al. 2001; Li et al. 2002; Ramanathan and Rosales-Hain 2000; Wattenhofer et al. 2001] on network topology control assumed that wireless ad hoc or sensor networks are modelled by *unit disk graphs* (UDG), *i.e.*, two mobile hosts can communicate as long as their Euclidean distance is no more than a threshold. However, practically, wireless ad hoc networks cannot be perfectly modelled as UDGs: the maximum transmission ranges of wireless devices may vary due to various reasons such as the device differences and the small mechanic/electronic errors during the process of transmitting even the transmission powers of all devices are set the same initially. In [Barriere et al. 2001; Kuhn and Zollinger 2003], the authors extended UDG into a new model, called *quasi unit disk graphs*, which is closer to reality than UDG. In this paper, we study a more generalized model. Each wireless node $u$ may have its own transmission radius $r_u$. Then heterogeneous wireless networks are modelled by mutual inclusion graphs (MG): two nodes can communicate directly only if they are within the transmission range of each other, *i.e.*, it has a physical link $uv$ if and only if $\|uv\| \leq \min(r_u, r_v)$. Clearly UDG is a special case of MG. Obviously, in the MG graph model for heterogeneous wireless ad hoc networks, a link $uv$ is always symmetric, *i.e.*, $u$ and $v$ can communicate directly with each other. We adopt this symmetric communication model since uni-directed links in wireless ad hoc networks are shown to be costly [Prakash 1999]. The topology control for wireless networks modelled by UDG has been investigated by a considerable amount of research efforts. The topology control for heterogeneous networks is even harder, since many properties in homogeneous networks disappear in heterogeneous networks. Thus, we cannot simply extend the ideas from the well-studied topologies, such as GG, RNG and Yao, used in homogeneous networks to heterogeneous wireless networks.

The main contributions of this paper are as follows. We present several localized topology control strategies in which every wireless sensor maintains communications to only a selected small subset of its physical neighbors in heterogeneous network environment. Here an algorithm is said to construct a topology $H$ *locally* if, every node $u$ can decide which edges $uv$ belong to $H$ using only the information of nodes within a constant number of hops of $u$. We prove that the global logical network topologies formed by these locally selected links are sparse and/or power efficient and our methods are communication efficient. Here a structure is power efficient if the total power consumption of the least cost path connecting any two nodes in it is no more than a small constant factor of that in the original heterogeneous communication network. By utilizing the wireless broadcast channel capability, and assuming that a message sent by a sensor node will be received by all sensors within its transmission region with at most a constant number of transmissions, we prove that all our methods use at most $O(n)$ total messages, where each message has $O(\log n)$ bits. By further assuming that the transmission ranges of two neighboring nodes are within a constant $\gamma$ factor of each other (such wireless network is called *smoothed*), we prove that some of the proposed structures also have a constant bounded logical node degree. We also conduct extensive simulations to study the practical performances of our methods.

The rest of the paper is organized as follows. In Section 2, we introduce the background and review previous methods. Limitations on heterogeneous network topology control are discussed in Section 3. We describe various localized methods in which individual wireless sensor nodes collectively form a sparse structure in Section 4, form a sparse power spanner in Section 5, and form a degree-bounded power and length spanner in Section 6. We also analyze the communication complexities of these methods. Our theoretical results are corroborated in the simulations in Section 7. We conclude our paper in Section 8 with the discussion of future works.

## 2. PRELIMINARIES

### 2.1 Heterogeneous Wireless Network Model

A heterogeneous wireless network, *e.g.*, wireless sensor networks, is composed of a set $V$ of $n$ wireless devices (called node hereafter) $v_1, v_2, \cdots, v_n$, in which each node $v_i$ has its own maximum transmission power $p'_i$. Let $\epsilon_i$ be the mechanic/electronic error of a node $v_i$ in its power control. Then the maximum transmission power considered in this paper is actually $p_i = p'_i - \epsilon$. We adopt a common assumption in the literature that the power needed to support the communication between two nodes $v_i$ and $v_j$ is $\|v_i v_j\|^\beta$, where $\beta \in [2, 5]$ is a real number depending on the environment and $\|v_i v_j\|$ is the Euclidean distance between $v_i$ and $v_j$. Consequently, the signal sent by a node $v_i$ can be received by all nodes $v_j$ with $\|v_i v_j\| \le r_i$, where $r_i^\beta \le p_i/p_0$, $p_0$ is the uniform threshold that a signal with power $p_0$ can be recognized by a node. [4] Thus, for simplicity, we assume that each mobile host $v_i$ has its own transmission range $r_i$. The heterogeneous wireless ad hoc network is then modelled by a mutual inclusion graph (MG), where two nodes $v_i$, $v_j$ are connected if and only if they are within the transmission range of each other, *i.e.*, $\|v_i v_j\| \le \min(r_i, r_j)$.

[4]Notice that, in practice, not all nodes within distance of $r_i$ of node $v_i$ can receive the signal sent by $v_i$. We adopt the model used here because of two reasons: first, it is the model that is widely used for the theoretical study of heterogeneous wireless ad hoc networks; second, using this model, we are able to show that our methods generate topologies with some nice properties.

Previously, only a few methods are known for topology control when the networks are modelled as mutual inclusion graphs.

In this paper, we also assume that each wireless node knows its geometry position either via GPS or some localization methods [Cheng et al. ; Hu and Evans 2004]. Notice that given the estimated geometry positions of all wireless nodes, we can derive the mutual inclusion communication network topology (called *derived network topology) (DMG)* which has a link $v_iv_j$ if and only if $\|\overline{v_i v_j}\| \leq \min(r_i, r_j)$. Here $\overline{v_i}$ denotes the estimated geometry position of the wireless node $v_i$. In this paper, we assume that the structure DMG is same as the physical communication network MG even under the presence of the geometry position error (*i.e.*, the distance $\|\overline{v_i}v_i\|$ between estimated location $\overline{v_i}$ and its physical location $v_i$) by a localization method. The correctness of our methods do not require that each wireless node knows the exact geometry position information. But on the other hand, the spanner properties proved for our methods do depend on the the localization precision.

### 2.2 Current State of Knowledge

Many structures were proposed for topology control in homogeneous wireless ad hoc networks. Due to limited spaces, we will briefly review some of proximity geometric structures. The *relative neighborhood graph* [Toussaint 1980] $RNG(V)$ consists of all edges $uv$ such that the intersection of two circles centered at $u$ and $v$ and with radius $\|uv\|$ do not contain any vertex $w$ from $V$. The *Gabriel graph* [Gabriel and Sokal 1969] $GG(V)$ contains edge $uv$ if and only if $disk(u, v)$ contains no other vertices of $V$, where $disk(u, v)$ is the disk with edge $uv$ as a diameter. Both $GG(V)$ and $RNG(V)$ are connected, planar, and contain the Euclidean minimum spanning tree of $V$. The intersections of $GG(V)$, $RNG(V)$ with a connected $UDG(V)$ are connected. Delaunay triangulation, denoted by $Del(V)$, is also used as underlying structure by several routing protocols. Here a triangle $\triangle uvw$ belongs to $Del(V)$ if its circumcircle does not contain any node inside. It is well known that $RNG(V) \subseteq GG(V) \subseteq Del(V)$. The intersection of $Del(V)$ with a connected $UDG(V)$ has a bounded length spanning ratio [Li et al. 2002].

The *Yao graph* [Yao 1982] with an integer parameter $k \geq 6$, denoted by $\overrightarrow{YG}_k(V)$, is defined as follows. At each node $u$, any $k$ equally-separated rays originated at $u$ define $k$ cones. In each cone, choose the shortest edge $uv$ among all edges from $u$, if there is any, and add a directed link $\overrightarrow{uv}$. Ties are broken arbitrarily or by ID. The resulting directed graph is called the *Yao graph.* Let $YG_k(V)$ be the undirected graph by ignoring the direction of each link in $\overrightarrow{YG}_k(V)$. Some researchers used a similar construction named $\theta$-graph [Lukovszki 1999], the difference is that it chooses the edge which has the shortest projection on the axis of each cone instead of the shortest edge in each cone.

The initial effort for topology control in heterogeneous wireless networks was reported in [Song et al. 2003] by the same authors of this paper. In [Song et al. 2003], we showed how to perform topology control based on Yao structure for heterogeneous wireless networks. Recently, several structures that extend the relative neighborhood graph and local minimum spanning tree were proposed in [Li et al. 2004] for topology control in heterogeneous wireless networks. They build directed network topologies while the methods presented here build undirected topologies that are beneficial for routing. Since their original methods cannot preserve the network connectivity, two structures were proposed in the online version of their paper [Li et al. 2004]: an extended relative neighborhood graph and an extended local minimum spanning tree.

### 2.3 Spanners and Stretch Factors

When constructing a subgraph of the original communication network MG, we may need consume more power to connect some nodes since we may disconnect the most power efficient path connecting them in MG. Thus, naturally, we would require that the constructed structure approximates MG well in terms of the power consumption for unicast routing. In graph theoretical term, the structure should be a spanner [Arya et al. 1995; Lukovszki 1999]. Let $G = (V, E)$ be a $n$-vertex weighted connected graph. The distance in $G$ between two vertices $u, v \in V$ is the length of the shortest path between $u$ and $v$ and it is denoted by $d_G(u, v)$. A subgraph $H = (V, E')$, where $E' \subseteq E$, is a *t-spanner* of $G$ if for every $u, v \in V$, $d_H(u, v) \leq t \cdot d_G(u, v)$. The value of $t$ is called the *stretch factor* or *spanning ratio*. When the graph is a geometric graph and the weight is the Euclidean distance between two vertices, the stretch factor $t$ is called the *length stretch factor*, denoted by $\ell_H(G)$. For wireless networks, the mobile devices are usually powered by batteries only. We thus pay more attention to the power consumptions. When the weight of a link $uv \in G$ is defined as the power to support the communication of link $uv$, the stretch factor of $H$ is called the *power stretch factor*, denoted by $\rho_H(G)$ hereafter. The power, denoted by $p_G(u, v)$, needed to support the communication between a link $uv$ in $G$ is often assumed to be $\|uv\|^\beta$, where $2 \leq \beta \leq 5$. Obviously, for any weighted graph $G$ and a subgraph $H \subseteq G$, we have the following lemma (the detailed proof can be found in [Li et al. 2001]):

LEMMA 1. *[Li et al. 2001] Graph $H$ has stretch factor $\delta$ if and only if for any link $uv \in G$, $d_H(u, v) \leq \delta \cdot d_G(u, v)$.*

Thus, to generate a spanner $H$ with spanning ratio $\rho$, we only have to make sure that *every link $uv$* of $G$ is approximated within a constant factor $\rho$: there is a path connecting $u$ and $v$ in $H$ with weight at most $\rho$ times the weight of $uv$.

### 2.4 Sparseness and Bounded Degree

All well-known proximity graphs ($GG(V)$, $RNG(V)$, $Del(V)$ and $YG(V)$) have been proved to be sparse graphs when network is modelled as a UDG. Recall that a *sparse* graph means the number of edges is linear with the number of nodes. The sparseness of all well-known proximity graphs implies that the average node degree[5] is bounded by a constant. Moreover, we prefer the maximum node degree is bounded by a constant, because wireless nodes have limited resources and a large number of communication neighbors often implies huge signal interference in wireless communications. In addition, unbounded degree (or in-degree) at a node

[5]Throughout this paper, we use *node degree* as an alternative of *logical* node degree, *i.e.*, the number of selected neighbors, unless we explicitly use *physical* node degree to denote the number of nodes within transmission radius. Similarly, without causing confusion, when we talk about links, we always mean *logical* links.

$u$ will often cause large overhead at $u$, whereas a bounded degree increases the network throughput. In addition, bounded degree will also give us advantages when apply several routing algorithms. Therefore, it is often imperative to construct a sparse network topology with a bounded node degree while it is still power-efficient. However, Li *et al.* [Li et al. 2001] showed that the maximum node degree of RNG, GG and Yao could be as large as $n-1$. The instance consists of $n-1$ nodes $v_i$ lying on the unit circle centered at a node $u \in V$. Then each edge $uv_i$ belongs to the $RNG(V)$, $GG(V)$ and $\overrightarrow{YG}_k(V)$.

Recently, in homogeneous wireless ad hoc networks, some improved or combined proximity graphs [Wang and Li 2003; Song et al. 2004] have been proposed to build planar degree-bounded power spanner topology, which meets all preferred properties for unicast. In heterogeneous networks, only a few research efforts [Li et al. 2004; Song et al. 2003] are reported so far. In the following, we will first discuss the difficulties and limitations for topology control in heterogeneous networks, then present our localized strategies in detail.

## 3. LIMITATIONS

In heterogeneous wireless ad hoc networks, a connected planar topology[6] does not necessarily exist. Figure 1 (a) shows an example. There are four nodes $x$, $y$, $u$ and $v$ in the network, where their transmission ranges $r_x = r_y = \|xy\|$ and $r_u = r_v = \|uv\|$, and node $u$ is out of the transmission range of node $x$ and $y$, while node $v$ is in the transmission range of node $y$ and out of the range of $x$. The transmission ranges of $x$ and $y$ are illustrated by the dotted circles. According to the definition of $MG$, there are only three edges $xy$, $vy$ and $uv$ in the symmetric communication graph. Hence any topology control method can not make the topology planar while keeping the communication graph connected. On the other hand, it is worth to think whether we can design a new routing protocol on some pseudo-planar topologies. As will see later, the pseudo-planar topology GG(MG) and RNG(MG) proposed in this section has some special properties which are different from other general non-planar topologies. For instance, two intersecting triangles can not share a common edge. We leave it as a future work to further investigate them.

Another limitation for topology control in heterogeneous networks is that the node degree can not be bounded by a constant if the ratio of the transmission radii of two neighboring nodes is unbounded. Figure 1 (b) shows such an example. In the example, a node $v$ has $p+1$ incoming neighbors $w_i$, $0 \leq i \leq p$. Assume that each node $w_i$ has a transmission radius $r_{w_i} = r_v/3^{p-i}$ and $\|vw_i\| = r_{w_i}$. Here $r_v$ is the transmission range of node $v$. Obviously, $\|w_iw_j\| > \min(r_{w_i}, r_{w_j})$, *i.e.*, any two nodes $w_i, w_j$ are not directly connected in MG. Then, none of those edges incident on $v$ can be deleted, hence there is no topology control method that can bound the

[6] Here, we call a geometry structure *planar topology* if there are no physical edge intersections in the geometric structure. It is different with the standard definition of *planar graph* which allows re-drawing the graph on a plane without crossing edges. One of the reasons that we use the special definition here is in wireless ad hoc networks some geometric routing algorithms use the real geometric positions for routing, and they ask for the underline geometric structure to be a planar topology without moving the node or using curved edges. Notice that using standard definition of planar, every connected graph has a planar subgraph, *e.g.*, a spanning tree.

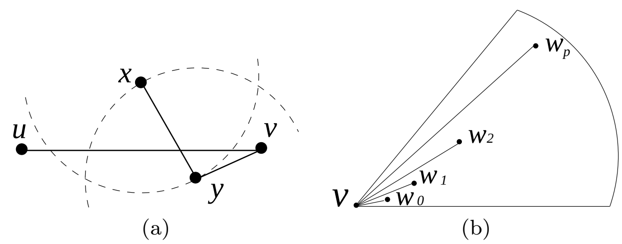


Fig. 1. Limitations on heterogeneous networks: (a) Planar topology does not exist. (b) Degree of node $v$ can not be bounded by constant.

node degree by a constant without violating the connectivity. Consider the example illustrated by Figure 1 (b), edges $vw_i$, $0 \leq i \leq p$, are all possible communication links. Thus, node $v$ in any connected spanning graph has degree $p+1$. On the other hand, we will show in section 6, in the worst case, any connected MG graph has degree $O(\log_2 \gamma)$ where $\gamma = \max_{v \in V} \max_{w \in I(v)} \frac{r_v}{r_w}$. Here, $I(v) = \{w \mid wv \in MG\}$. In the example, recall that $3^p r_{w_0} = r_v$, hence $\gamma$ equals to $3^p$ for this example. Thus, $v$ has degree $\log_3 \gamma + 1 = O(\log_2 \gamma)$. In the paper, we always assume $\gamma$ is a constant. It is practical, since two wireless devices in a nearby region often have similar transmission ranges. Generally, we call a wireless network *smoothed* if $\gamma$ is a constant for this network.

## 4. HETEROGENEOUS SPARSE STRUCTURE

In this section, we propose a strategy for all nodes to self-form a sparse structure, called $RNG(MG)$, based on the relative neighborhood graph. We will prove that the total number of links of this structure is $O(n)$.

DEFINITION 1 STRUCTURE $RNG(MG)$. *A link $uv \in MG$ is kept in $RNG(MG)$ if and only if there is* no *another node $w$ inside lune$(u, v)$ and* both *links $uw$ and $wv$ are in MG. Here lune$(u, v)$ is the intersection of disk$(u, \|uv\|)$ and disk$(v, \|uv\|)$.*

The construction algorithm will be similar to Algorithm 2 later, thus we omit it here. Notice that the total communication cost of constructing $RNG(MG)$ is $O(n \log n)$ bits, assuming that the radius and ID information of a node can be represented in $O(\log n)$ bits. In addition, the structure $RNG(MG)$ is symmetric: if a node $u$ keeps a link $uv$, node $v$ will also keep the link $uv$. Thus, a node $u$ does not have to tell its neighbor $v$ whether it keeps a link $uv$ or not.

It is not difficult to prove that structure $RNG(MG)$ is connected by induction. On the other hand, same as the case in homogeneous networks (*i.e.*, UDG mode), $RNG(MG)$ does not have a bounded length stretch factor, nor a constant bounded power stretch factor, and does not have a bounded node degree. In this paper, we will show that $RNG(MG)$ is a *sparse* graph: it has at most $6n$ links.

In the following, we define a new structure, called $ERNG(MG)$, and present a localized algorithm to construct it.

DEFINITION 2 STRUCTURE $ERNG(MG)$. *Each node $u$ keeps the link to neighbor $v \in B(u)$ if and only if there is* no *another node $w \in B(u)$ inside lune$(u, v)$ and* both *links $uw$ and $wv$ are in MG. Here $B(u) = \{v \mid r_v \geq r_u$ and $uv \in MG\}$*

*and lune(u, v) is the intersection of disk(u, ‖uv‖) and disk(v, ‖uv‖). All the links kept by all nodes form the final structure ERNG(MG).*

**Algorithm 1** Constructing-ERNG

1: Each node $u$ initiates sets $E_{MG}(u)$ and $E_{ERNG}(u)$ to be empty. Here $E_{MG}(u)$ is the set of links of MG known to $u$ so far and $E_{ERNG}(u)$ is the set of links of ERNG known to $u$ so far.
2: Then, each node $u$ locally broadcasts a Hello message with $ID_u$, $r_u$ and its position $(x_u, y_u)$ to all nodes within its transmission range. Note that $r_u = {p_u}^{1/\beta}$ is its maximum transmission range.
3: **while** node $u$ receives a Hello message from some node $v$ **do**
4: If $\|vu\| \leq \min\{r_u, r_v\}$, then node $u$ adds a link $uv$ to $E_{MG}(u)$.
5: If $r_v \geq r_u$, then node $u$ performs the following procedures. Node $u$ checks if there is another link $uw \in E_{MG}(u)$ with the following additional properties: 1) $w \in \mathit{lune}(u, v)$, 2) $r_w \geq r_u$, and 3) $\|wv\| \leq \min\{r_w, r_v\}$. If no such link $uw$, then add $uv$ to $E_{ERNG}(u)$.
6: For any link $uw \in E_{ERNG}(u)$, node $u$ checks if the following conditions hold: 1) $v \in \mathit{lune}(u, w)$, and 2) $\|wv\| \leq \min\{r_w, r_v\}$. If the conditions hold, then remove link $uw$ from $E_{ERNG}(u)$.
7: **end while**
8: For each link $uv \in E_{ERNG}(u)$, node $u$ informs node $v$ to add link $uv$.
9: The final topology is the undirected graph formed by the union of all links in $E_{ERNG}(u)$ for each node $u$ (ignoring the direction), and is called $ERNG(MG)$ .

We then prove the following lemma.

Lemma 2. *Structure ERNG(MG) has at most* $6n$ *links.*

Proof. Consider any node $u$. We will show that $u$ keeps at most 6 directed links $uv$, with $r_v \geq r_u$, emanated from $u$. Assume that $u$ keeps more than 6 directed links. Obviously, there are two links $uw$ and $uv$ such that $\angle wuv < \pi/3$. Thus, $vw$ is not the longest link in triangle $\triangle uvw$. Without loss of generality, we assume that $\|uw\|$ is the longest in triangle $\triangle uvw$. Notice that the existence of link $uw$ implies that $\|uw\| \leq \min(r_u, r_w) = r_u$. Consequently, $\|vw\| \leq \|uw\| \leq \min(r_u, r_w)$. Thus, from the fact that $r_u \leq r_v$, we know $\|vw\| \leq \min(r_v, r_w)$. Hence, link $vw$ does exist in the original communication graph MG. It implies that link $uw$ cannot be selected to ERNG. In other words, structure ERNG(MG) has at most $6n$ links. □

Similar to Lemma 2, we can prove the following lemma.

Lemma 3. *Structure RNG(MG) has at most* $6n$ *links.*

Proof. Imagine that each link $uv \in RNG(MG)$ has a direction as follows: $\overrightarrow{uv}$ if $r_u \leq r_v$. Then similar to Lemma 2, we can prove that each node $u$ only keeps at most 6 such imagined direct links. Thus, there are at most $6n$ links in RNG(MG). □

## 5. HETEROGENEOUS POWER SPANNER

In the previous section, we defined two structures based on the relative neighborhood graph. These structures are sparse, however, theoretically they could have arbitrary large power spanning ratio. In this section, we give a strategy for all nodes to self-form a power spanner structure, called $GG(MG)$, based on the Gabriel graph.

DEFINITION 3 STRUCTURE $GG(MG)$. *A link $uv \in MG$ is kept in $GG(MG)$ if and only if there is* no *another node $w$ inside $disk(u,v)$ and both links $uw$ and $wv$ are in MG.*

Our localized construction method works as follows.

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**Algorithm 2** Constructing-GG

1: Let $E_{MG}(u)$ and $E_{GG}(u)$ be the set of links known to $u$ so far from MG and GG respectively. Each node $u$ initiates both $E_{MG}(u)$ and $E_{GG}(u)$ as empty.
2: Then, each node $u$ locally broadcasts a HELLO message with $ID_u$, $r_u$ and its position $(x_u, y_u)$ to all nodes within its transmission range.
3: **while** node $u$ receives a HELLO message from some node $v$ **do**
4: If $\|vu\| \leq \min\{r_u, r_v\}$, then node $u$ adds a link $uv$ to $E_{MG}(u)$.
5: Node $u$ checks if there is another link $uw \in E_{MG}(u)$ with the following two additional properties: 1) $w \in disk(u,v)$, and 2) $\|wv\| \leq \min\{r_w, r_v\}$. If there is no such link $uw$, add $uv$ to $E_{GG}(u)$.
6: For any link $uw \in E_{GG}(u)$, node $u$ checks if the following two properties hold: 1) $v \in disk(u,w)$, and 2) $\|wv\| \leq \min\{r_w, r_v\}$. If they hold, remove link $uw$ from $E_{GG}(u)$.
7: **end while**
8: The final topology is the undirected graph formed by the union of all links in $E_{GG}(u)$ for each node $u$ (ignoring the direction), and is called $GG(MG)$ .

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We first show that Algorithm 2 builds the structure $GG(MG)$ correctly. For any link $uv \in GG(MG)$, clearly, we cannot remove them in Algorithm 2. For a link $uv \notin GG(MG)$, assume that a node $w$ is inside $disk(u,v)$ and both links $uw$ and $wv$ belong to MG. If node $u$ gets the message from $w$ first, and then gets message from $v$, clearly, $uv$ cannot be added to $E_{GG}(u)$. If node $u$ gets the message from $v$ first, then $u$ will remove $uv$ from $E_{GG}(u)$ (if it is there) when $u$ gets the information of $w$.

It is not difficult to prove by induction that structure $GG(MG)$ is connected if original network is connected. In addition, since we remove a link $uv$ only if there are two links $uw$ and $wv$ with $w$ inside $disk(u,v)$, it is easy to show that the power stretch factor of $GG(MG)$ is 1. In other words, the minimum power consumption path for any two nodes $v_i$ and $v_j$ in MG is still kept in $GG(MG)$. Remember that here we assume the power needed to support a link $uv$ is $\|uv\|^\beta$, for $\beta \in [2,5]$.

Similar as the structure $ERNG(MG)$, we can define a structure called $EGG(MG)$:

DEFINITION 4 STRUCTURE $EGG(MG)$. *A link $uv \in MG$ is kept in $EGG(MG)$ if and only if there is* no *another node $w$ inside $disk(u,v)$ such that $r_u \leq r_w$.*

On the other hand, same as the case in homogeneous networks (*i.e.*, UDG mode), $GG(MG)$ and $EGG(MG)$ are not length spanners, and do not have bounded node degree. Furthermore, it is unknown whether they are *sparse* graphs. Recently, it was proven in [Kapoor and Li 2003] that $GG(MG)$ has at most $O(n^{8/5}\log\gamma)$ edges where $\gamma = \max r_u/r_v$.

Notice that, the extension from Gabriel graph is non-trivial. In [Kapoor and Li 2003], two structures defined as follows even cannot guarantee the connectivity. In the first structure, called $LGG_0(MG)$, they remove a link $uv \in MG$ if there is another node $w$ inside $disk(u, v)$. In the second structure, called $LGG_1(MG)$, they remove a link $uv \in MG$ if there is another node $w$ inside $disk(u, v)$, and either link $uw$ or link $wv$ is in MG.

## 6. HETEROGENEOUS DEGREE-BOUNDED SPANNER

Undoubtedly, as described in preliminaries, we always prefer a structure has more nice properties, such as degree-bounded (stronger than sparse), power spanner etc. Naturally, we could extend the previous known degree-bounded spanner, such as the Yao related structures, from homogeneous networks to heterogeneous networks. Unfortunately, a simple extension of the Yao structure from UDG to MG even does not guarantee the connectivity. Figure 2 illustrates such an example. Here $r_u = r_v = \|uv\|$, $r_w = \|uw\|$, $r_x = \|vx\|$, and $\|uw\| < \|uv\|$, $\|uw\| < \|vw\|$, $\|vx\| < \|uv\|$, and $\|vx\| < \|ux\|$. In addition, $v$ and $w$ are in the same cone of node $u$, and nodes $x$ and $u$ are in the same cone of node $v$. Thus, the original MG graph contains links $uv$, $uw$ and $vx$ only and is connected. However, when applying Yao structure on all nodes, node $u$ will only have information of node $v$ and $w$ and it will keep link $uw$. Similarly, node $w$ keeps link $uw$; node $v$ keeps link $vx$; and node $x$ keeps link $xv$. In other words, only link $xv$ and $uw$ are kept by Yao method. Thus applying Yao structure disconnects node $v$, $x$ from the other two nodes $u$ and $w$. Consequently, we need more sophisticated extensions of the Yao structure to MG to guarantee the connectivity of the structure.

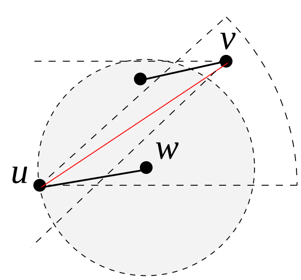



Fig. 2. Simple extension of Yao structure does not guarantee the connectivity.

### 6.1 Sparse Spanner

In our first sparse spanner structure $EYG_k(MG)$, unlike traditional Yao structure, each node $u$ keeps a node $v$ as communication neighbor if and only if $uv$ is the shortest link among links between $u$ and all nodes $v_i$ in the same cone of $u$, and $r_{v_i} \geq r_u$. Formally speaking, it is defined as follows.

DEFINITION 5 STRUCTURE $EYG_k(MG)$. *Each node $u$ partitions its transmission region into $k$ equal-sized cones. In each cone, it keeps a communication neighbor $v$ if $v$ is the closest neighbor among all nodes $v_i$ such that $r_{v_i} \geq r_u$ in the cone. Let $\overrightarrow{EYG}_k(MG)$ be the union of all chosen links. The undirected graph by ignoring the direction of each link in $\overrightarrow{EYG}_k(MG)$ is called $EYG_k(MG)$.*

Notice that, since node $u$ chooses a node $v \in disk(u, r_u)$ with $r_v \geq r_u$, link $uv$ is indeed a bidirectional link, *i.e.*, $u$ and $v$ are within the transmission range of each other. Additionally, as will see later, this strategy avoids the possible disconnection by simple Yao extension we mentioned before. The localized construction algorithm is as follows:

**Algorithm 3** Constructing-EYG

1: Initially, each node $u$ divides the disk $disk(u, r_u)$ centered at $u$ with radius $r_u$ by $k$ equal-sized cones centered at $u$. We generally assume that the cone is half open and half-close. Let $\mathbb{C}_i(u)$, $1 \leq i \leq k$, be the $k$ cones partitioned. Let $C_i(u)$, $1 \leq i \leq k$, be the set of nodes $v$ inside the $i$th cone $\mathbb{C}_i(u)$ with a *larger or equal* radius than $u$. In other words,

$$C_i(u) = \{v \mid v \in \mathbb{C}_i(u),\ and\ r_v \geq r_u\}.$$

Initially, $C_i(u)$ is empty.

2: Each node $u$ broadcasts a HELLO message with $ID_u$, $r_u$ and its position $(x_u, y_u)$ to all nodes in its transmission range.

3: **while** node $u$ receives a HELLO message from some node $v$ **do**

4: Node $u$ sets $C_i(u) = C_i(u) \bigcup \{v\}$, if node $v$ is inside the $i$th cone $\mathbb{C}_i(u)$ of node $u$ and $r_v \geq r_u$.

5: Node $u$ chooses a node $v$ from each cone $C_i(u)$ such that the link $uv$ has the smallest $ID(uv)$ among all links $uv_j$ with $v_j$ in $C_i(u)$, if there is any.

6: **end while**

7: Finally, each node $u$ informs all 1-hop neighbors of its chosen links through a broadcast message. Let $\overrightarrow{EYG}_k(MG)$ be the union of all chosen links. The final topology is the undirected graph by ignoring the direction of each link in $\overrightarrow{EYG}_k(MG)$, and is called $EYG_k(MG)$.

In the algorithm, each node only broadcasts twice: one for broadcasting its ID, radius and position; and the other for broadcasting the selected neighbors. Remember that it selects at most $k$ neighbors. Thus, each node sends messages at most $O((k+1) \cdot \log n)$ bits. Here, we assume that the node ID and its position can be represented using $O(\log n)$ bits for a network with $n$ wireless nodes. Obviously, we also have the following lemma:

[6]This is the main difference between this algorithm and the simple extension of Yao structure discussed before, in which it considers all nodes $v$ that $u$ can get signal from.

Lemma 4. *Structure $EYG_k(MG)$ has at most kn links where $k > 6$ is a constant.*

Before we study other properties of this structure, we have to define some terms first. Assume that each node $v_i$ of MG has a unique identification number $ID_{v_i} = i$. The identity of a bidirectional link $uv$ is defined as $ID(uv) = (\|uv\|, ID_u, ID_v)$ where $ID_u > ID_v$. Note that we use the bidirectional links instead of the directional links in the final topology to guarantee connectivity. In other words, we require that both node $u$ and node $v$ can communicate with each other through this link. In this paper, all proofs about connectivity or stretch factors take the notation $uv$ and $vu$ as same, which is meaningful. Only in the topology construction algorithm or proofs about bounded-degree, $uv$ is different from $vu$: the former is initiated and built by $u$, whereas the latter is by node $v$. Sometimes we denote a directional link from $v$ to $u$ as $\overrightarrow{vu}$ if necessary. Then we can order all bidirectional links (at most $n(n-1)$ such links) in an increasing order of their identities. Here the identities of two links are ordered based on the following rule: $ID(uv) > ID(pq)$ if (1) $\|uv\| > \|pq\|$ or (2) $\|uv\| = \|pq\|$ and $ID_u > ID_p$ or (3) $\|uv\| = \|pq\|$, $u = p$ and $ID_v > ID_q$.

Correspondingly, the rank of each link $uv$, denoted by $rank(uv)$, is its order in sorted bidirectional links. Notice that, we actually only have to consider the links in MG. We then show that the constructed network topology $EYG_k(MG)$ is a length and power spanner.

Theorem 5. *The length stretch factor of $EYG_k(MG)$, $k > 6$, is at most $\ell = \frac{1}{1-2\sin(\frac{\pi}{k})}$.*

Proof. Notice it is sufficient to show that for any nodes $u$ and $v$ with $\|uv\| \leq \min(r_u, r_v)$, *i.e.*, $uv \in MG$, there is a path connecting $u$ and $v$ in $EYG_k(MG)$ with length at most $\ell\|uv\|$. We construct a path $u \leftrightsquigarrow v$ connecting $u$ and $v$ in $EYG_k(MG)$ as follows.

Assume that $r_u \leq r_v$. If link $uv \in EYG_k(MG)$, then set the path $u \leftrightsquigarrow v$ as the link $uv$. Otherwise, consider the $disk(u, r_u)$ of node $u$. Clearly, node $u$ will get information of $v$ from $v$ and node $v$ will be selected to some $C_i(u)$ since $r_v \geq r_u$. Thus, from $uv \notin EYG_k(MG)$, there must exist another node $w$ in the same cone as $v$, which is a neighbor of $u$ in $EYG_k(MG)$. Then set $u \leftrightsquigarrow v$ as the concatenation of the link $uw$ and the path $w \leftrightsquigarrow v$. Here the existence of path $w \leftrightsquigarrow v$ can be easily proved by induction on the distance of two nodes. Notice that the angle $\theta$ of each cone section is $\frac{2\pi}{k}$. When $k > 6$, then $\theta < \frac{\pi}{3}$. It is easy to show that $\|wv\| < \|uv\|$. Consequently, the path $u \leftrightsquigarrow v$ is a simple path, *i.e.*, each node appears at most once.

We then prove by induction that the path $u \leftrightsquigarrow v$ has total length at most $\ell\|uv\|$.

Obviously, if there is only one edge in $u \leftrightsquigarrow v$, $d(u \leftrightsquigarrow v) = \|uv\| < \ell\|uv\|$. Assume that the claim is true for any path with $l$ edges. Then consider a path $u \leftrightsquigarrow v$ with $l+1$ edges, which is the concatenation of edge $uw$ and the path[7] $w \leftrightsquigarrow v$ with $l$ edges, as shown in Figure 3 where $\|wv\| = \|xv\|$.

[7] In the procedure of induction, if $r_w \leq r_v$ then we induct on path $w \leftrightsquigarrow v$, otherwise we induct on path $v \leftrightsquigarrow w$. In fact, here $w \leftrightsquigarrow v$ is same as $v \leftrightsquigarrow w$ since the path is bidirectional for communication. Directional link is only considered in building process and is meaningless when we talk about the path. This induction rule is applied throughout the remainder of the paper.

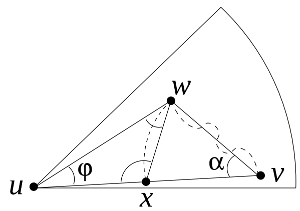



Fig. 3. The length stretch factor of $EYG_k(MG)$ is at most $\frac{1}{1-2\sin(\frac{\pi}{k})}$.

By induction, $d(w \leftrightsquigarrow v) \le \ell\|wv\|$. Let $\varphi = \angle wuv$ and $\alpha = \angle uvw$, then

$$\frac{\|uw\|}{\|ux\|} = \frac{\sin(\angle uxw)}{\sin(\angle xwu)} = \frac{\sin(\pi - \angle wxv)}{\sin(\varphi + \angle uxw)} = \frac{\sin(\pi - \frac{\pi-\alpha}{2})}{\sin(\varphi + \pi - \frac{\pi-\alpha}{2})} = \frac{\sin(\frac{\pi}{2} + \frac{\alpha}{2})}{\sin(\frac{\pi}{2} + \frac{\alpha}{2} + \varphi)}$$

$$= \frac{1}{\cos\varphi - \sin\varphi \tan\frac{\alpha}{2}} \le \frac{cos(\frac{\pi}{4} - \frac{\varphi}{4})}{cos(\frac{\pi}{4} + \frac{3}{4}\varphi)} \le \frac{1}{1 - 2\sin(\frac{\pi}{k})}$$

The first inequality is due to $0 \le \alpha \le \frac{\pi}{2} - \frac{\varphi}{2}$ and the second inequality is due to $0 \le \varphi \le \frac{2\pi}{k}$. Consequently, $d(u \leftrightsquigarrow v) = \|uw\| + d(w \leftrightsquigarrow v) < \ell\|ux\| + \ell\|wv\| = \ell\|uv\|$, where $\ell = \frac{1}{1-2\sin(\frac{\pi}{k})}$. That is to say, the claim is also true for the path $u \leftrightsquigarrow v$ with $l+1$ edges.

Thus, the length stretch factor of $EYG_k(MG)$ is at most $\ell = \frac{1}{1-2\sin(\frac{\pi}{k})}$. □

THEOREM 6. *The power stretch factor of structure $EYG_k(MG)$, $k > 6$, is at most* $\rho = \frac{1}{1-(2\sin\frac{\pi}{k})^\beta}$.

PROOF. The proof is similar to that in UDG [Li et al. 2001; Li et al. 2002] except the induction procedure. We show by induction, on the number of its edges, that the path $u \leftrightsquigarrow v$ constructed in Theorem 5 has power cost, denoted by $p(u \leftrightsquigarrow v)$, at most $\rho\|uv\|^\beta$. □

### 6.2 Novel Space Partition

Partitioning the space surrounding a node into $k$ equal-sized cones enables us to bound the node out-degree using the Yao structure. Using the same space partition, Yao-Yao structure [Li et al. 2001; Li et al. 2002] produces a topology with bounded in-degree when the networks are modelled by UDG. Yao-Yao structure (for UDG) is generated as follows: a node $u$ collects all its incoming neighbors $v$ (*i.e.*, $\overrightarrow{vu} \in \overrightarrow{YG}_k(V)$), and then selects the closest neighbor $v$ in each cone $\mathbb{C}_i(u)$. Clearly, Yao-Yao has bounded degree at most $k$. They also showed that another structure YaoSink [Li et al. 2001; Li et al. 2002] has not only the bounded node degree but also a constant bounded stretch factor. The network topology with a bounded degree can increase the communication efficiency. However, these methods [Li et al. 2001; Li et al. 2002] may fail when the networks are modelled by MG: they cannot even guarantee the connectivity, which is verified by following discussions.

Assume that we already construct a connected directed structure $\overrightarrow{EYG}_k(MG)$. Let $I(v) = \{w \mid \overrightarrow{wv} \in \overrightarrow{EYG}_k(MG)\}$. In other words, $I(v)$ is the set of nodes that have directed links to $v$ in $\overrightarrow{EYG}_k(MG)$. Let $I_i(v) = I(v) \cap \mathbb{C}_i(u)$, *i.e.*, the nodes in

$I(v)$ located inside the $i$th cone $\mathbb{C}_i(v)$. Yao-Yao structures will pick the closest node $w$ in $I_i(v)$ and add undirected link $wv$ to Yao-Yao structure. Previous example in Figure 1 (b) also illustrates the situation that Yao-Yao structure is not connected. In the example, a node $v$ has $p+1$ incoming neighbors $w_i$, $0 \le i \le p$. Assume that each node $w_i$ has a transmission radius $r_{w_i} = r_v/3^{p-i}$ and $\|vw_i\| = r_{w_i}$. Obviously, $\|w_iw_j\| > \min(r_{w_i}, r_{w_j})$, *i.e.*, any two nodes $w_i, w_j$ are not directly connected in MG. It is easy to show that the Yao structure $\overrightarrow{EYG}_k(MG)$ only has directed links $\overrightarrow{w_iv}$. Obviously, node $v$ will only select the closest neighbor $w_0$ to the Yao-Yao structure, which disconnects the network. This same example can also show that the structure based on Yao-Sink [Li et al. 2001; Li et al. 2002] is also not connected for heterogeneous wireless ad hoc networks.

Thus, selecting the closest incoming neighbor in each cone $\mathbb{C}_i$ is too aggressive to guarantee the connectivity. Observe that, in Figure 1 (b), to guarantee the connectivity, when we delete a directed link $\overrightarrow{w_iv}$, we need to keep *some* link, say $w_jv$, such that $w_iw_j$ is a link in MG. Thus, we further partition the cone into a limited number of smaller *regions* and we will keep *only* one node in each region, *e.g.*, the closest node. Clearly, to guarantee that other nodes in the same region are still connected to $v$, we need make sure that any two nodes $w_i$, $w_j \in I(v)$ that co-exist in a same small region are directly connected in MG. Consequently, if the number of regions is bounded by a constant, a degree-bounded structure could be generated. In the remainder of this subsection, we will introduce a novel space partition strategy satisfying the above requirement.

For each node $v$, let $\gamma_v = \max_{w \in I(v)} \frac{r_v}{r_w}$. Remember that all nodes in $I(v)$ have transmission radius at most $r_v$, so $\gamma_v \ge 1$. Let $h$ be the positive integer satisfying $2^{h-2} < \gamma_v \le 2^{h-1}$. Our proposed partition method works as follows.

METHOD 1. *Partition Transmission Disks*

1: *Each node $v$ divides each cone centered at $v$ into a limited number of triangles and caps, as illustrated by Figure 4, where $\|va_i\| = \|vb_i\| = \frac{1}{2^{h-i}} r_v$ and $c_i$ is the mid-point of the segment $a_ib_i$, for $1 \le i \le h$.*

2: *The triangles $\triangle va_1b_1$, $\triangle a_ib_ic_{i+1}$, $\triangle a_ia_{i+1}c_{i+1}$, $\triangle b_ib_{i+1}c_{i+1}$, for $1 \le i \le h-1$, and the cap $\widehat{a_nb_n}$ form the final space partition of each cone. For simplicity, we call such a triangle or the cap as a* region.

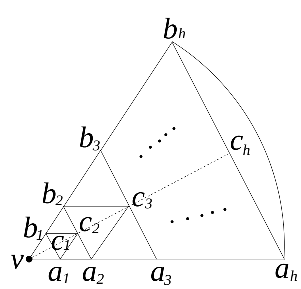



Fig. 4. Extend Yao structure on heterogeneous networks: Further space partition in each cone to bound in-degree.

We then prove that this partition indeed guarantees that any two nodes in any same region are connected in MG.

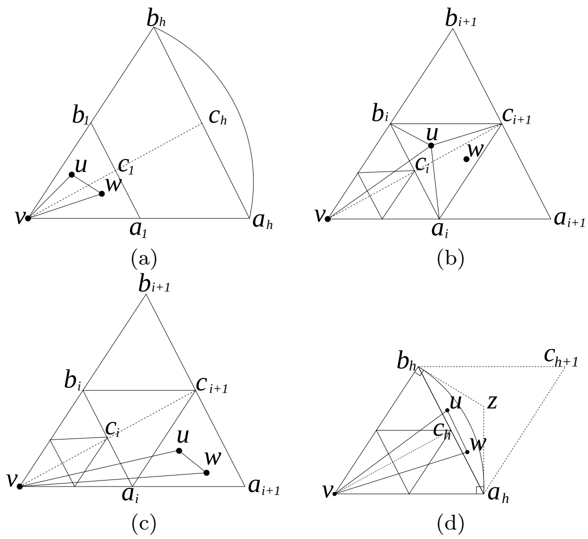



Fig. 5. (a) Two nodes are in triangle $\triangle va_1b_1$. (b) Two nodes are in triangle $\triangle a_ib_ic_{i+1}$. (c) Two nodes are in triangle $\triangle a_ia_{i+1}c_{i+1}$. (d) Two nodes are inside cap $\widehat{a_hb_h}$.

LEMMA 7. *Assume that $k \geq 6$. Any two nodes $u, w \in I(v)$ that co-exist in any one of the generated regions are directly connected in MG, i.e., $\|uw\| < \min(r_u, r_w)$.*

PROOF. We prove this lemma based on the region where these two nodes are located. There are four different cases.

(1) Two nodes are in $\triangle va_1b_1$, as shown in Figure 5 (a).
Remember that all nodes in $I(v)$ have transmission radius at least $\|va_1\| = \frac{1}{2^{(h-1)}} r_v$. We have $\min(r_u, r_w) \geq \|va_1\| = \|vb_1\|$ and $\|a_1b_1\| \leq \|va_1\|$. In addition, since $uw$ is a segment inside $\triangle va_1b_1$, we have $\|uw\| \leq \max(\|a_1b_1\|, \|va_1\|, \|vb_1\|)$. Consequently, $\|uw\| < \min(r_u, r_w)$, *i.e.* $uw \in MG$.

(2) Two nodes are in $\triangle a_ib_ic_{i+1}$, as shown in Figure 5 (b).
In this case, we have
   (a) $\|vu\| > \|uc_{i+1}\|$, since $a_ib_i$ is the perpendicular bisector of $vc_{i+1}$ and $u$ is at the same side of line $a_ib_i$ as $c_{i+1}$.
   (b) $\|vu\| > \|ua_i\|$, because $\angle va_iu > \frac{\pi}{3} > \angle uva_i$.
   (c) $\|vu\| > \|ub_i\|$, because $\angle vb_iu > \frac{\pi}{3} > \angle uvb_i$.
   (d) $\|uw\| < \max(\|uc_{i+1}\|, \|ua_i\|, \|ub_i\|)$, because node $w$ must be inside one of the triangles $\triangle a_ib_iu$, $\triangle a_ic_{i+1}u$ and $\triangle b_ic_{i+1}u$.

Thus, $\|uw\| < \|uv\|$. Similarly, $\|uw\| < \|wv\|$. Consequently, $uw \in MG$ from

$$\|uw\| < \min(\|uv\|, \|wv\|) < \min(r_u, r_w).$$

(3) Two nodes are in $\triangle a_i a_{i+1} c_{i+1}$, as shown in Figure 5 (c). We have $\min(r_u, r_w) \geq \|va_i\| = \|a_i a_{i+1}\| = \|a_i c_{i+1}\| > \|a_{i+1} c_{i+1}\|$. Since $uw$ is a segment inside $\triangle a_i a_{i+1} c_{i+1}$, $\|uw\| < \max(\|a_i a_{i+1}\|, \|a_i c_{i+1}\|, \|a_{i+1} c_{i+1}\|) \leq \min(r_u, r_w)$, *i.e.*, $uw \in MG$. Triangle $\triangle b_i b_{i+1} c_{i+1}$ is the symmetric case with triangle $\triangle a_i a_{i+1} c_{i+1}$, so the claim holds similarly.

(4) Two nodes are inside the cap $\widehat{a_h b_h}$, as shown in Figure 5 (d), where $a_h z$ and $b_h z$ is the tangent of arc $\widehat{a_h b_h}$ at point $a_h$ and $b_h$ respectively.
Since $\angle a_h v b_h < \frac{2\pi}{k}$, $k \geq 6$, we have

$$\angle v b_h z = \frac{\pi}{2} < \pi - \angle a_h v b_h = \angle v b_h c_{h+1}.$$

Similarly, $\angle v a_h z < \angle v a_h c_{h+1}$. This means $\widehat{a_h b_h}$ is inside $\triangle a_h b_h c_{h+1}$. The remaining of the proof directly follows from the proof for the case of $\triangle a_i b_i c_{i+1}$.

This finishes the proof. □

### 6.3 Bounded Degree Sparse Structure

Using the space partition discussed in Section 6.2, we present our method to locally build a sparse network topology with bounded degree for heterogeneous wireless sensor network. Here we assume that $\gamma = \max_{v \in V} \gamma_v$ is bounded by some constant, where $\gamma_v = \max_{w \in I(v)} \frac{r_v}{r_w}$, and $I(v) = \{w \mid \overrightarrow{wv} \in \overrightarrow{EYG}_k(MG)\}$.

Definition 6 Structure $EYY_k(MG)$. *A link $\overrightarrow{uv}$ is kept by a node $v$ if $u$ is the closest neighbor in the corresponding region of $\overrightarrow{EYG}_k(MG)$ that $u$ locates at. The union of all chosen links is the final network topology, denoted by $\overrightarrow{EYY}_k(MG)$. We call it* Extended Yao-Yao *graph. The structure $EYY_k(MG)$ is the undirected graph by ignoring the direction of each link in $\overrightarrow{EYY}_k(MG)$.*

Algorithm 4 illustrates our method constructing the structure $EYY_k(MG)$ in a localized way.

**Algorithm 4** Constructing-EYY

1: Each node finds the incident edges in the structure $\overrightarrow{EYG}_k(MG)$, as described in Algorithm 3.
2: Each node $v$ partitions the $k$ cones centered at $v$ using the partitioning method described in Method 1. Notice that for partitioning, node $v$ uses parameter $\gamma_v$ in Method 1, which can be easily calculated from local information. Figure 6 (a) illustrates such a partition.
3: Each node $v$ chooses a node $u$ from each generated region so that the link $\overrightarrow{uv}$ has the smallest $ID(uv)$ among all directed links toward to $v$ computed in the first step in the partition. Figure 6 (b) illustrates such a selection of incoming links.
4: Finally, for each link $uv$ selected by $v$, node $v$ informs node $u$ of keeping link $uv$. Let $\overrightarrow{EYY}_k(MG)$ be the union of all chosen links. The final topology is the undirected graph by ignoring the direction of each link in $\overrightarrow{EYY}_k(MG)$, and is called $EYY_k(MG)$.

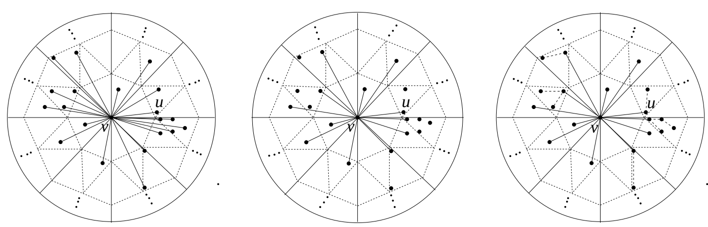
THEOREM 8. *The out-degree of each node $v$ in $\overrightarrow{EYY}_k(MG)$, $k \geq 6$, is bounded by $k$ and the in-degree is bounded by $(3\lceil \log_2 \gamma_v \rceil + 2)k$, where $\gamma_v = \max_{w \in I(v)}(\frac{r_v}{r_w})$.*

PROOF. It is obvious that the out-degree of a node $v$ is bounded by $k$ because the out-degree bound of $\overrightarrow{EYG}_k(MG)$ is $k$ and Algorithm 4 does not add any directed link.

For the in-degree bound, as shown in Figure 4, obviously, the number of triangle regions in each cone is $3h-2$. Remember that $2^{h-2} < \gamma_v \leq 2^{h-1}$, which implies $h = 1 + \lceil \log_2 \gamma_v \rceil$. Thus, considering the cap region also, the in-degree of node $v$ is at most $(3\lceil \log_2 \gamma_v \rceil + 2)k$. □

Let $\gamma = \max_v \gamma_v$. Obviously, the maximum node degree in graph $EYY_k(MG)$ is bounded by $(3\lceil \log_2 \gamma \rceil + 3)k$.

Notice that the structure $EYY_k(MG)$ is a subgraph of the structure $EYG_k(MG)$, thus, there are at most $k \cdot n$ edges in $EYY_k(MG)$. Consequently, the total communications of Algorithm 4 is at most $O(k \cdot n)$, where each message has $O(\log n)$ bits. It is interesting to see that the communication complexity does not depend on $\gamma$ at all.




(a) (b) (c)

Fig. 6. (a) In $EYG_k(MG)$, star formed by links toward to $v$. (b) Node $v$ chooses the shortest link in $EYG_k(MG)$ toward itself from each region to produce $EYY_k(MG)$. (c) The sink structure at $v$ in $EYY_k^*(MG)$.

THEOREM 9. *The graph $EYY_k(MG)$, $k \geq 6$, is connected if MG is connected .*

PROOF. Notice that it is sufficient to show that there is a path from $u$ to $v$ for any two nodes with $uv \in MG$. Remember the graph $EYG_k(MG)$ is connected, therefore, we only have to show that $\forall uv \in EYG_k(MG)$, there is a path connecting $u$ and $v$ in $EYY_k(MG)$. We prove this claim by induction on the ranks of all links in $EYG_k(MG)$.

If the link $uv$ has the smallest rank among all links of $EYG_k(MG)$, then $uv$ will obviously survive after the second step. So the claim is true for the smallest rank.

Assume that the claim is true for all links in $EYG_k(MG)$ with rank at most $r$. Then consider a link $uv$ in $EYG_k(V)$ with $rank(uv) = r+1$ in $EYG_k(MG)$. If $uv$ survives in Algorithm 4, then the claim holds. Otherwise, assume that $r_u < r_v$. Then directed edge $vu$ cannot belong to $\overrightarrow{EYG}_k(MG)$ from Algorithm 3. Thus, directed edge $uv$ is in $\overrightarrow{EYG}_k(MG)$. In Algorithm 4, directed edge $uv$ can only be removed by node $v$ due to the existence of another directed link $wv$ with a smaller identity and $w$ is in the same region as $u$. In addition, the angle $\angle wvu$ is less than

$\theta = \frac{2\pi}{k} (k \geq 6)$. Therefore we have $\|wu\| < \|uv\|$. Notice that here $wu$ is guaranteed to be a link in MG, but it is not guaranteed to be in $EYG_k(MG)$. We then prove by induction that there is a path connecting $w$ and $u$ in $EYY_k(MG)$. Assume $r_w \leq r_u$. The scenario $r_w > r_u$ can be proved similarly. There are two cases here.

Case 1: the link $wu$ is in $EYG_k(MG)$. Notice that rank of $wu$ is less than the rank of $uv$. Then by induction, there is a path $w \leftrightsquigarrow u$ connecting $w$ and $u$ in $EYY_k(MG)$. Consequently, there is a path (concatenation of the undirected path $w \leftrightsquigarrow u$ and the link $wv$) between $u$ and $v$.

Case 2: the link $wu$ is not in $EYG_k(MG)$. Then, from proof of Theorem 5, we know that there is a path $\Pi_{EYG_k}(w, u) = q_1 q_2 \cdots q_m$ from $w$ to $u$ in $EYG_k(MG)$, where $q_1 = w$ and $q_m = u$. Additionally, we can show that each link $q_i q_{i+1}$, $1 \leq i < m$, has a smaller rank than $wu$, which is at most $r$. Here $rank(q_1 q_2 = wq_2) < rank(w, u)$ because the selection method in Algorithm 3. And $rank(q_i q_{i+1}) < rank(w, u)$, $1 < i < m$, because

$$\|q_i q_{i+1}\| \leq \|q_i u\| < \|q_{i-1} u\| < \cdots < \|q_1 u\| = \|wu\|.$$

Then, by induction, for each link $q_i q_{i+1}$, there is a path $q_i \leftrightsquigarrow q_{i+1}$ survived in $EYY_k(MG)$ after Algorithm 4. The concatenation of all such paths $q_i \leftrightsquigarrow q_{i+1}$, $1 \leq i < m$, and the link $wv$ forms a path from $u$ to $v$ in $EYY_k(MG)$. □

Although $EYY_k(MG)$ is a connected structure, it is unknown whether it is a power or length spanner. We leave it as a future work.

### 6.4 Bounded Degree Sparse Spanner

In [Li et al. 2001; Li et al. 2002], the authors applied the technique in [Arya et al. 1995] to construct a sparse network topology in UDG, *Yao and sink graph*, which has a bounded degree and a bounded stretch factor. The technique is to replace the directed star in Yao graph consisting of all links toward a node $v$ by a directed tree $T(v)$ with $v$ as the sink. Tree $T(v)$ is constructed recursively. To apply this technique on MG, we need a more sophisticated way to guarantee the connectivity. In the remainder of this section, we discuss how to locally construct a bounded degree structure with bounded power stretch factor for heterogeneous wireless sensor networks. Our method works as follows.

Notice that, sink node $v$, not $u$, constructs the tree $T(u)$ and then informs the end-nodes of the selected links to keep such links if already exist or add such links otherwise.

Notice that Algorithm 6 is only performed by a node $v$ where $u$ is some incoming neighbor of $v$ in $\overrightarrow{EYG}_k(MG)$. We then prove that the constructed structure $\overrightarrow{EYG}^*_k(MG)$ indeed has bounded degree (thus sparse), and is power efficient.

THEOREM 10. *The maximum node degree of the graph $\overrightarrow{EYG}^*_k(MG)$ is at most $k^2 + 3k + 3k \cdot \lceil \log_2 \gamma \rceil$.*

PROOF. Initially, each node has at most $k$ out-degrees after constructing graph $EYG_k(MG)$. In the algorithm, each node $v$ initiates only one sink structure, which will introduce at most $(3\lceil \log_2 \gamma \rceil + 2) \cdot k$ in-degrees. Additionally, each node $x$ will be involved in Algorithm 6 for at most $k$ sink trees (once for each directed link $xy \in EYG_k(MG)$). For each sink tree involvement, Algorithm 6 assigns at most $k$

**Algorithm 5** Constructing-EYG*

1: Each node finds the incident edges in the structure $\overrightarrow{EYG}_k(MG)$, as described in Algorithm 3. Each node $v$ will have a set of incoming neighbors $I(v) = \{u \mid \overrightarrow{uv} \in \overrightarrow{EYG}_k(MG)\}$.
2: Each node $v$ partitions the $k$ cones centered at $v$ using the partitioning method described in Method 1. Notice that for partitioning, node $v$ uses parameter $\gamma_v$ in Method 1, which can be easily calculated from local information. Figure 6(a) illustrates such a partition.
3: Each node $v$ chooses a node $u$ from each region $\Omega$. Let $\Omega_u(v)$ be the region $\Omega$ partitioned by node $v$ inside which node $u$ locates. Node $u$ is chosen such that the link $uv$ has the smallest $ID(uv)$ among all links computed in the first step in the region $\Omega_u(v)$. In other words, in this step, it constructs $\overrightarrow{EYY_k(MG)}$.
4: For each region $\Omega_u(v)$ and the selected node $u$, let $S_\Omega(u) = \{w \mid w \neq u,\ w \in \Omega_u(v) \cap I(v)\}$, *i.e.*, the set of incoming neighbors of $v$ (other than $u$) in the same region as $u$. For each node $u$, node $v$ uses the following function $\mathsf{Tree}(u, S_\Omega(u))$ (described in Algorithm 6) to build a tree $T(u)$ rooted at $u$. We call $T(u)$ a *sink tree* and call the union of all links chosen by node $v$ the *sink structure* at $v$. Figure 6(c) illustrates a sink structure at $v$, which is composed of all trees $T(u)$ for $u$ selected in the previous step.
5: Finally, node $v$ informs nodes $x$ and $y$ for each selected link $xy$ in the sink structure rooted at $v$.
6: Let $\overrightarrow{EYG}^*_k(MG)$ be the union of all chosen links. The final topology is the undirected graph by ignoring the direction of each link in $\overrightarrow{EYG}^*_k(MG)$, and is called $EYG^*_k(MG)$.

**Algorithm 6** Construct Tree($u$,$S_\Omega(u)$)

1: If $S_\Omega(u)$ is empty, then return.
2: Otherwise, partition the disk centered at $u$ by $k$ equal-sized cones: $\mathbb{C}_1(u)$, $\mathbb{C}_2(u)$, $\cdots$, $\mathbb{C}_k(u)$.
3: Find the node $w_i \in S_\Omega(u) \cap \mathbb{C}_i(u)$, $1 \leq i \leq k$, with the smallest $ID(w_i u)$, if there is any. Link $w_i u$ is added to $T(u, S_\Omega(u))$ and node $w_i$ is removed from $S_\Omega(u)$.
4: For each node $w_i$, call $\mathsf{Tree}(w_i,\ S_\Omega(u) \cap \mathbb{C}_i(u))$ and add the created edges to $T(u, S_\Omega(u))$.

links incident on $x$. Thus, at most $k^2$ new degrees could be introduced here. Then the theorem follows. □

Notice that, it is not difficult to show that the total number of links added by a node $v$ is at most $|I(v)|$, *i.e.*, the number of its incoming neighbors. We already showed that the total number of directed links in $\overrightarrow{EYG}_k(MG)$ is at most $kn$. Thus, we have the following lemma:

LEMMA 11. *Total number of links in $EYG^*_k(MG)$ is at most $kn$.*

It also implies that the total communication cost of Algorithm 5 is $O(k \cdot n)$. Here each message has $O(\log n)$ bits.

THEOREM 12. *The length stretch factor of $EYG^*_k(MG)$, $k > 6$, is at most* $(\frac{1}{1-2\sin(\frac{\pi}{k})})^2$.

PROOF. We have proved that $EYG_k(MG)$ has length stretch factor at most $\frac{1}{1-2\sin(\frac{\pi}{k})}$. We thus have only to prove that, for each link $vw \in EYG_k(MG)$, there is a path connecting them in $EYG^*_k(MG)$ with length at most $\frac{1}{1-2\sin(\frac{\pi}{k})}\|vw\|$. If link $vw$ is kept in $EYG^*_k(MG)$, then this is obvious. Otherwise, assume $r_w \leq r_v$, then directed link $wv$ belongs to $\overrightarrow{EYG_k}(MG)$. Then, there must exist a node $u$ in the same region (partitioned by node $v$) as node $w$. Using the same argument as Theorem 5, we can prove that there is a path connecting $v$ and $w$ in $T(u)$ with length at most $\frac{1}{1-2\sin(\frac{\pi}{k})}\|vw\|$. It implies that the length stretch factor of $EYG^*_k(MG)$ is at most $(\frac{1}{1-2\sin(\frac{\pi}{k})})^2$. □

Similarly, we have:

THEOREM 13. *The power stretch factor of the graph $EYG^*_k(MG)$, $k > 6$, is at most* $(\frac{1}{1-(2\sin\frac{\pi}{k})^\beta})^2$.

## 7. SIMULATIONS

In this section we measure the performance of the proposed heterogeneous network topologies by conducting extensive simulations. In our simulations, we randomly generate a set $V$ of $n$ wireless nodes with random transmission range for each node. We then construct the mutual inclusion communication graph $MG(V)$, and test the connectivity of $MG(V)$. If it is connected, we construct different localized topologies: $GG(MG)$, $EGG(MG)$, $RNG(MG)$, $ERNG(MG)$, $EYG_k(MG)$, $EYY_k(MG)$ and $EYG^*_k(MG)$. Then we measure the sparseness (the average node degree), the power efficiency and the communication cost of building these topologies. In the simulation results presented here, the wireless nodes are distributed in a $400m \times 400m$ square field. Each wireless node has a transmission radius randomly selected from $[60m, 260m]$. The number of wireless nodes is $30i$, where $i$ is varied from 1 to 10. For each $1 \leq i \leq 10$, we randomly generate 100 sets of $30i$ nodes. All structures proposed in this paper are generated for each set of nodes. The number of cones is set to 7 for $EYG_k(MG)$, $EYY_k(MG)$ and $EYG^*_k(MG)$.

### 7.1 Node Degree

First of all, we test the sparseness of each network topology proposed in this paper. Notice that, we have theoretically proved that $RNG(MG)$ and $ERNG(MG)$ have at most $6n$ links; $EYG_k(MG)$ has at most $k \cdot n$ links, where $k \geq 7$ is the number of cones divided; $EYY_k(MG)$ also has at most $k \cdot n$ links since $EYY_k(MG) \subseteq EYG_k(MG)$; $EYG^*_k(MG)$ also has at most $k \cdot n$ links since the sink structure for each node $u$ has exactly the number of links as the number of directed links toward $u$ in the directed structure $\overrightarrow{EYG_k}(MG)$. We do not know how many links $GG(MG)$ and $EGG(MG)$ could have.

Although almost all proposed structures are sparse theoretically, all of them could have unbounded node degree. The node degree of the wireless networks should not be too large. Otherwise a node with a large degree has to communicate with many nodes directly. This potentially increases the signal interference and the overhead at this node. Figure 7 (a) illustrates the average node degree of different topologies. Notice that graph $RNG(MG)$ always has the smallest average node degree in our simulations and structure $EYG_k^*(MG)$ always has the largest average node degree. We also found that the average node degree becomes almost stable when the number of nodes increases, *i.e.*, the network becomes denser.

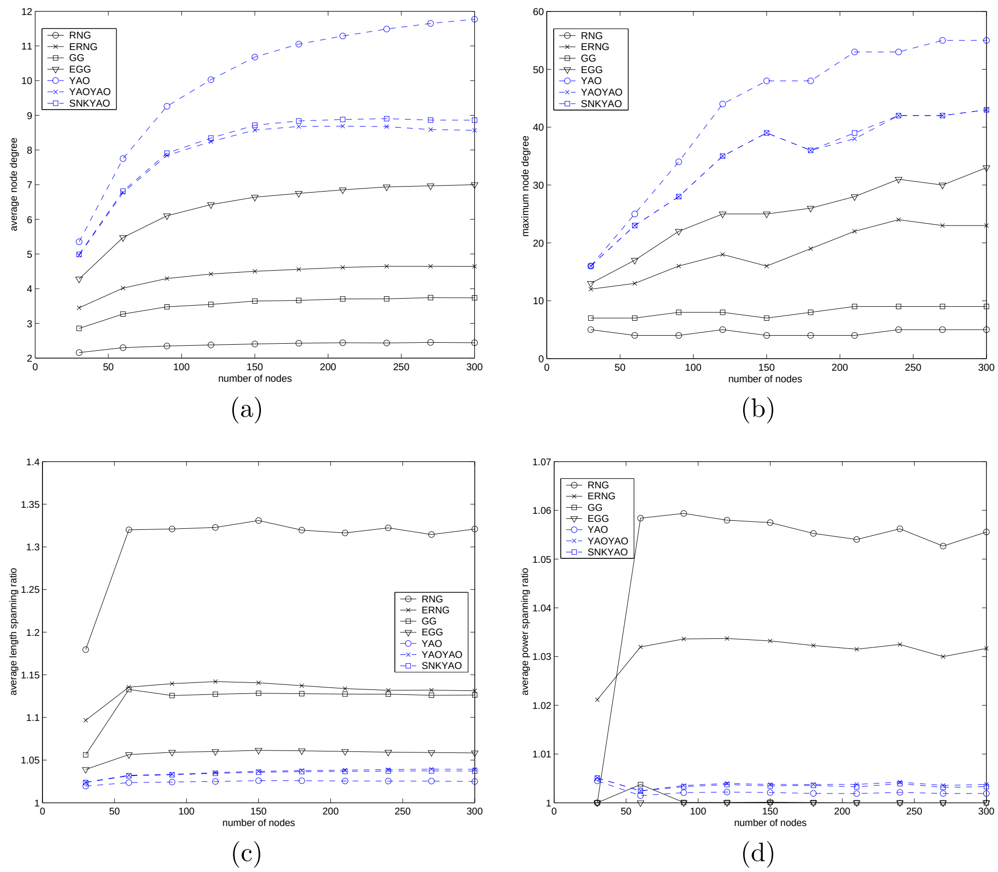



Fig. 7. (a) Average node degree of different topologies. (b) Maximum node degree of Yao-based structures. (c) Average length spanning ratio of different topologies. (d) Average power spanning ratio of different topologies.

Figure 7 (b), as proved in Theorem 10, confirms that the maximum node degree of Yao-based structure $EYG_k^*(MG)$ is bounded by $3k \cdot \log_2 \gamma + k^2 + 3k$, where $\gamma = \max_{uv \in MG} \frac{r_u}{r_v}$. This figure also shows that $EYG_k(MG)$ generally will have a maximum node degree larger than $EYG_k^*(MG)$ and $EYY_k(MG)$. It is interesting to see that the maximum degree of $EYG_k^*(MG)$ and $EYY_k(MG)$ almost have the same curve when network density changes. It also shows that $GG(MG)$ and

$RNG(MG)$ has smallest max-degree among all of them, though they do not have theoretical degree-bound. The reason is that the worst case example happens rare in random networks. In addition, as expected, $EGG(MG)$ and $ERNG(MG)$ keep more links then $GG(MG)$ and $RNG(MG)$, hence have bigger max-degree.

Given the size of the network $n = 30i$, we take the average of the maximums of all 100 random networks with $n$ nodes we generated as the final maximum value for $n$ plotted here.

### 7.2 Spanning Ratio

We proved that $GG(MG)$ and $EGG(MG)$ have power spanning ratios exactly one; $EYG_k(MG)$ and $EYG_k^*(MG)$ both have bounded length and power spanning ratios. Notice that $RNG(MG)$ and $ERNG(MG)$ could have power and length spanning ratios as large as $n-1$ for a network of $n$ nodes; and the length spanning ratios of $GG(MG)$ and $EGG(MG)$ could be $\sqrt{n-1}$ even when all nodes have the same transmission range. It is unknown whether $EYY_k(MG)$ has a bounded length or power spanning ratio even for wireless networks modelled by UDG. We then conduct extensive simulations to study how good these structures are for heterogeneous networks when the nodes' transmission ranges are randomly set.

Figure 7 (c) illustrates the length spanning ratios of these structures. As the theoretical results suggest, we found that $RNG(MG)$ has a much larger length spanning ratio compared with other structures. It is surprising to see that $ERNG(MG)$ also has a much smaller spanning ratio than $RNG(MG)$. We know that $ERNG(MG)$ has a smaller spanning ratio than $RNG(MG)$ since $ERNG(MG) \supseteq RNG(MG)$. Also notice that $EYG_k(MG)$, as the theoretical results suggest, has the smallest spanning ratio among all structures proposed here.

For wireless ad hoc networks, we want to keep as fewer links as possible while still keep relatively power efficient paths for every pair of nodes. Figure 7 (d) illustrates the power spanning ratios of these structures. Here we assume that the power needed to support a link $uv$ is $\|uv\|^2$. As we expected, structures $GG(MG)$ and $EGG(MG)$ keep the most power efficient path for every pair of nodes, *i.e.*, their power spanning ratios are exactly one. In our simulations, we found that $RNG(MG)$ and $ERNG(MG)$ indeed have the largest power spanning ratios among all proposed structures.

### 7.3 Communication Cost of Construction

It is not difficult to see that $GG(MG)$, $RNG(MG)$ can be constructed using only $n$ messages by assuming that each node can tell its neighbors its maximum transmission range, and its geometry position information in one single message. Each node $u$ can uniquely determine all the links $uv$ in these three structures after knowing all its one hop neighbors in $MG$. Structures $EYG_k(MG)$, $EYY_k(MG)$ and $EYG_k^*(MG)$ can be constructed using only $k \cdot n + n$ messages since the final structures have at most $kn$ links. Similarly, $ERNG(MG)$ can be constructed using at most $7n$ messages. We do not know any theoretical bound about the number of messages needed to construct $EGG(MG)$ since each node $u$ has to inform its neighbors the links selected by $u$ for $EGG(MG)$. We measured the actual average number of messages needed to construct these structures. We only measure the average number of messages per wireless node for $EGG(MG)$, $ERNG(MG)$, $EYG_k(MG)$, and $EYG_k^*(MG)$ (since every node only has to spend one message for other three structures $GG(MG)$, $RNG(MG$, and $EYG_k(MG)$). Figure 8 illustrates the communication cost. We found that structure $EYG_k^*(MG)$ is the most expensive one

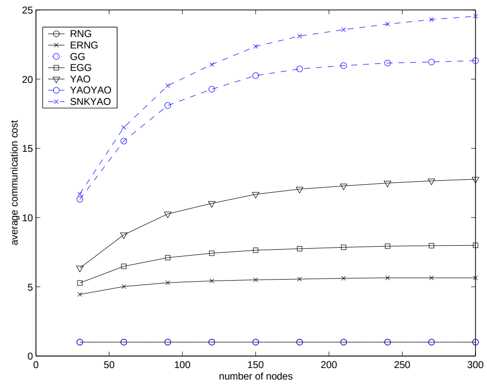



Fig. 8. Average communication cost of building different topologies.

to construct although it has the most favorable properties theoretically (bounded length, power spanning ratio and bounded node degree). Constructing $EYG_k^*(MG)$ is almost as expensive as constructing $EYG_k(MG)$.

## 8. CONCLUSION

In this paper, we studied topology control for heterogeneous wireless sensor networks, where wireless sensors may have different maximum transmission powers and two sensors are connected if and only if they are within the maximum transmission range of each other. We presented several strategies for all wireless sensor nodes self-maintaining sparse and power efficient topologies in heterogeneous network environment with low communication cost. Table I summarizes the differences of all those proposed structures. All structures $GG(MG)$, $RNG(MG)$, $EYG_k(MG)$, $EYY_k(MG)$, and $EYG_k^*(MG)$ are connected if MG is connected, while $EYG_k(MG)$ and $EYG_k^*(MG)$ have constant bounded power and length stretch factors. Additionally, we showed that $EYY_k(MG)$ and $EYG_k^*(MG)$ have bounded node degrees $O(\log_2 \gamma)$, here $\gamma = \max_{v \in V} \max_{w \in I(v)}(\frac{r_v}{r_w})$. In the worst cast any connected graph will have degree at least $O(\log_2 \gamma)$ for heterogeneous wireless sensor networks. In other words, the structures constructed by our methods are almost optimum in terms of the minimum logical node degree. Our algorithms are all localized and have communication cost at most $O(n)$, where each message has $O(\log n)$ bits.

It remains an open problem whether graph $EYY_k(MG)$ is a length or power spanner. It is also unknown how many links $GG(MG)$ could have in the worst case although we show that it is definitely less than $O(n^{8/5} \log_2 \gamma)$[Kapoor and Li 2003]. Some other future works are what are the conditions that we can build a structure with some other properties for MG, such as planar or low weight. Notice that we can not build a pseudo-planar topology for an arbitrary heterogeneous wireless

Table I. The performances comparison of structures for heterogeneous networks.

| | Sparse | Spanner | Bounded Degree | Communication Cost |
|---|---|---|---|---|
| $RNG(MG)$ | Yes | No | No | O(n) |
| $ERNG(MG)$ | Yes | No | No | O(n) |
| $GG(MG)$ | Unknown | Power Spanner | No | O(n) |
| $EGG(MG)$ | Unknown | Power Spanner | No | Unknown |
| $EYG_k(MG)$ | Yes | Length and Power Spanner | No | O((k+1)n) |
| $EYY_k(MG)$ | Yes | Unknown | Yes | O((k+1)n) |
| $EYG_k^*(MG)$ | Yes | Length and Power Spanner | Yes | O((k+1)n) |

sensor network as showed in the paper, but it is unknown whether we can build such planar structure if some reasonable constraints are applied.

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