Waveform Inversion Assisted Distributed Reverse-Time Migration for Microseismic Location

Fangyu Li, Yan Qin, and WenZhan Song

Abstract—We present a novel approach to locate microseismic sources in-situ and in real time in distributed sensor networks. We propose a distributed reverse-time migration (RTM) microseismic source location algorithm. RTM based methods have advantages in passive source location in terms of robustness and accuracy. However, the traditional methods have a centralized data collection and ex-situ post-processing style, and were not designed for in-situ and real time seismic imaging, so communication and computation costs were not considered. Utilizing newly emerging sensor networks, real time and in-situ microseismic source location becomes possible. Thus, we specially design a joint imaging condition for the distributed sensor network system to reduce both computation and communication burdens. The tradeoff, however, is location resolution reduction. Sequentially, we employ a waveform inversion approach to obtain a finer resolution microseismic source location result. Finally, we validate the proposed method using both synthetic and field seismic data. The proposed waveform inversion assisted distributed RTM location algorithm obtains high resolution source location results and significantly reduced communication and computation costs, even at a low signal to noise ratio.

Index Terms—reverse-time migration, distributed sensor network, microseismic source location, joint imaging condition.

I. INTRODUCTION

NATURAL and human activities can induce underground seismic activity [1], [2]. To characterize subsurface structures and monitor underground activities, source location is necessary for analyzing all kinds of different scale activities [3], such as, volcano monitoring [4], unconventional reservoirs interpretation [5], injection monitoring [6], fracture zone characterization [7], etc. Usually, earthquakes with magnitude not bigger than 2.0 are called microearthquakes, whose corresponding sources are naturally named microseismic sources.

Commonly, an arrival time difference based method is used for passive source localization [8], which requires arrival picking based on the seismic waveform properties [9]–[11]. Due to the waveform dependence, a low signal-to-noise ratio (SNR) may affect the arrival picking accuracy and location stability. In contrast, the reverse time migration (RTM) technique applies the principle of time-reversal invariance of wavefields, which enables optimal focusing of the source locations [12], [13]. Via various imaging conditions, the migrated wavefields are combined constructively and deconstructively to characterize the underground structures and activities [14], [15]. Based on the joint imaging conditions, RTM is a robust imaging technique showing promising noise immunity [16]. To speed up the calculation, the RTM can be implemented in a parallelized manner [17], and a 3D-to-2D data conversion filter could be applied in order to reduce the calculation cost [18].

Fig. 1: Microseismic source location using sensor networks.

Except the well-planned unconventional resource development monitoring, the centralized processing and computing style is not capable of being implemented in situ and real time subsurface imaging in all circumstances, especially in harsh environments [19]. For example, wirelessly connected sensors were deployed using an air-dropped way to monitor live volcano activities, where communication and computation become bottlenecks [20]. Recently, seismic tomography has been implemented using advanced wireless sensor networks with distributed computing algorithms [21]–[23]. The distributed style has advantages in reducing the data loss risk in case of node and cable failures, because the sensing, computing and data storage tasks can be operated in the sensor nodes. Instead of collecting data into a processing center, distributed seismic data processing and computing can be performed on individual sensors with communications among the local sensor array.

Various imaging conditions for passive source location have been proposed for RTM [18]. In [14], the possibility to use RTM in the sensor networks was investigated. Although a distributed RTM microseismic location algorithm was proposed, the communication cost was not considered. For real seismic sensor network applications, we propose a novel distributed imaging condition to further reduce the communication and computation costs. Nevertheless, the communication and computation reduction comes with a cost that the resulting resolution might be decreased. To improve the imaging resolution, the least-squares RTM is commonly used in exploration geophysics [24], while
waveform inversion (WI) is extensively used in seismology to provide improved velocity models and source locations [25]. In this paper, we present a real-time in-situ microseismic source location method using distributed sensor networks. Fig. 1 demonstrates the application of our method, where in-situ and real time microseismic imaging is obtained based on distributed sensor networks. We develop a WI assisted distributed RTM algorithm designed for sensor networks. This is a milestone for underground activity monitoring, especially for geologically active areas with volcanic activities or unconventional resources developments. Our contributions are:

1) Distributed microseismic location algorithm is designed for real sensor networks. Joint imaging condition and distributed boundary condition are adopted to reduce computation and communication costs;
2) WI based high resolution post-processing is employed to improve the microseismic source location accuracy, which is important for underground activity analysis.

The remainder of this paper is organized as follows. In Section II, we introduce the theoretical principles of RTM, microseismic source location, WI and joint imaging conditions. In Section III, we describe the proposed algorithm in detail. We also analyze the computation/communication complexity of our algorithm, compared with the centralized RTM and the previous distributed RTM [14] algorithms. Then, synthetic experiment results are presented to evaluate the performance of the proposed method in terms of image quality and robustness in Section IV. Later, a field application example is shown in Section V. Finally, conclusions are drawn in Section VI.

II. THEORY

A. Reverse Time Migration

Assuming seismic wave propagation from a source location \(x_s\) to a receiver location \(x_r\), the recorded waveform \(W_r(x, t)\) at time \(t\), as shown in Fig. 2(a), can be expressed in a convolution format. (Note that the common wave conversion format is in Appendix A.) Based on the Green’s function \(G(x_r, t, x_s)\) between the source and receiver, and source wavefield \(W_S(x, t)\) generated from the source location, the receiver waveform \(W_r(x, t)\) is given as \(W_r(x, t) = W_S(x, t) \odot G(x_r, t, x_s)\). If we propagate the recorded wavefields \(W_r\) in the reverse time, which means the wavefield will propagate starting from the receiver location \(x_r\), it is called time-reversal imaging.

A “movie” \(I\) at location \(x\) recorded by receiver \(r\):

\[
I(x, t) = W_S(x, t) \odot W_r(x, t),
\]

where \(W_r(x, t)\) is the time reverse wavefield at receiver \(r\) and \(W_S(x, t)\) is still the source wavefield, which can be approximated using a finite-difference solution.

B. Microseismic Source Location Using RTM

Fig. 2(b) shows the principle to use wave back propagation to locate the seismic source. Wavefields \(W_{r_i}(x, t)\) and \(W_{r_j}(x, t)\) are generated by the same seismic source. For simplicity, we assume that the source function is a delta function and consider the reciprocity property \(G(x_{r_i}, t, x_s) = G(x_s, t, x_{r_i})\), such that the convolution function between the observed data at receivers \(r_i\) and \(r_j\) is \(\{G(x_{r_i}, t, x_s)G(x_{r_j}, t, x_s)\}\). Thus, the recorded waveform at receiver \(r_i\) can be inferred from that at receiver \(r_j\): \(W_{r_i}(x, t) = G(x_{r_i}, t, x_s)W_{r_j}(x, t)\).

Consider the receiver \(r_j\) as a virtual source \(W_{r_j}(x, t)\), applying an active-shot RTM to the convolution function, we obtain an image of the virtual scatterer (i.e., seismic source):

\[
I(x, t) = W_{r_i}(x, t) \odot W_{r_i}(x, t)
\]

\[
= \{G(x_{r_i}, t, x_s)G(x_{r_j}, t, x_s) \odot W_{r_j}(x, t)\} \odot W_{r_j}(x, t)
\]

\[
= \{G(x_{r_i}, t, x_s) \odot W_{r_j}(x, t)\} \{G(x_{r_j}, t, x_s) \odot W_{r_j}(x, t)\}
\]

(2)

where, the complex conjugate of the virtual-source wavefields \(W_{r_i}(x, x_{r_i})\) is related to the fact that the imaging condition is given by cross-correlation [17]. Because the entire observed wavefield \(W_{r_i}(x, t)\) is used, time-reversal imaging does not require arrival-time picking. Therefore, we can use time-reversal imaging for microseismic data with low SNR.

C. Joint Imaging Condition

In a centralized manner, it is typical to collect all the data to a centralized computing facility and calculate the wavefields. However, implementing RTM in a sensor network manner requires considering the communication costs [14].

Existing imaging conditions: Typically, without considering the communication cost, the imaging condition can be formulated as a multiplication of all wavefields \(W_{r_n}\) obtained from each receiver (the total number \(N\)): \(I(x, t) = \prod_N W_{r_n}(x, t)\), or the summation of all wavefields: \(I(x, t) = \sum_N W_{r_n}(x, t)\). The major differences between these two imaging conditions are the resolution, amplitude preservation and computational costs. Typically, the multiplication has higher resolution, while...
the summation preserves the amplitude and is more robust. As well, the multiplication is computationally heavier than the summation [14].

A GmRTM (geometric mean product RTM) imaging condition is proposed to collapse the time axis [15]: \( I(x) = \sum_T \prod_{N} W_{r_i}(x, t) \). However, as a centralized RTM method, GmRTM requires solving the wave equation independently for each receiver, which can be computationally demanding. Grouping several receivers and solving the extrapolation problem for each group (shown in Fig. 3), in [14], a hybrid cross-correlation time-reversal imaging condition was proposed:

\[
I(x) = \sum_{T} \sum_{K} \prod_{M} W_{r_k,m}(x, t),
\]

where, the \( N \) receivers are divided into \( K \) clusters, and each cluster has \( M \) receivers, so \( N = K \times M \). Note that the wavefields need to be transmitted over the network for imaging.

Proposed imaging condition: From Eq. (3), although computation cost has been reduced by \( M \) times, the communication cost is still high since the 4D wavefields need to be transmitted among the sensors (receivers). Thus, we propose a novel imaging condition especially designed for distributed sensor network:

\[
I(x) = \sum_{K} \sum_{T} \prod_{M} W_{r_k,m}(x, t).
\]

We can define a time window to segment the continuum time series seismic data into short sequences and refresh the image after processing one sequence. Compared with [14], it is clear that instead of transmitting the wavefields among the different clusters, we only transfer the seismic data within the same cluster and then combine the stacked images from all the clusters into a final energy map. Then the microseismic hypocenter location is the position, where and when all the wavefields coincide. The computation and communication reduction is significant, which will be discussed in Section III.

D. Waveform Inversion

Adopting the joint distributed imaging condition, we can save communication and computation cost. However, this comes at a cost of image resolution reduction. To improve the location image resolution, we propose a waveform inversion step following the distributed RTM. Traditionally, the waveform inversion is associated with velocity model update [16]. Because our goal is to propose a in-situ and real time seismic source location method in a distributed sensor network, we only update the location in our study for the sake of efficiency.

The forward modelling process can be simplified as \( d = Gf \), where \( d = [d_1, d_2, ..., d_N]^T \) represents the seismic recordings, \( G \) represents forward modeling operator including source injection, finite difference propagation and data acquisition. \( f = [f_1, f_2, ..., f_N]^T \) is the unknown vector of sources.

The source location problem is solved via regularized least squares inversion:

\[
\arg \min_{m \in M} \| d - Gf \|^2 + \lambda \| L_{x,z,t} \|_{1,1,2},
\]

where \( M \) denotes the energy concentrated area, and the norm \( \| \cdot \|_{1,1,2} \) permits one to estimate solutions that are sparse in space and smooth in time. Notice that the final source location will not be a single point but a more concentrated area than the direct results from distributed RTM depending on the regularization term and the optimization stopping criterion.

III. WI ASSISTED DISTRIBUTED RTM SOURCE LOCATION SYSTEM DESIGN

Based on the theories introduced and proposed in the previous section, we propose a distributed microseismic source location algorithm based on WI and distributed RTM. In our study, the source location is determined based on the source possibility map, on which stronger energy means higher probability of the underground activities. The detailed system workflow can be found in Algorithm 1.

Algorithm 1 The proposed microseismic location system

| Input: Raw seismic data. |
| Output: Microseismic location results. |
| 1: Locally cluster sensor nodes. |
| 2: Data transmission in the cluster. |
| 3: Use the sensors in the cluster as the boundary condition to generate the partial energy map. |
| 4: Transmit data among sink nodes from different clusters in the whole network. |
| 5: Apply the proposed imaging condition to generate the whole energy map. |
| 6: Select an AOI (area of interest) to improve the efficiency. |
| 7: WI on the AOI to improve the resolution. |

A main contribution of the proposed system is the computation and communication cost reduction. Our distributed architecture is shown as Fig. 3, where filled dots and hollow dots are computational nodes in the network. The hollow dots are normal nodes. The sensor network is separated into several clusters, each of which has one sink node (the filled dot). The sink nodes collect seismic data from its neighbors and apply the local RTM algorithm presented in Algorithm 1. The solid lines in Fig. 3 denote data transmission within the clusters; while the dashed lines indicate the partial source location result transmission among sink nodes in the whole network. In the end, WI is performed in the sink nodes.

The essential difference between our algorithm and previous distributed RTM method [14] is that our algorithm stacks RTM images in the time domain first, then transmits the local images.
TABLE I: Communication and computation cost analysis.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Communication Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centralized RTM</td>
<td>$T \times N \times F_s$</td>
</tr>
<tr>
<td>Distributed RTM</td>
<td>$t_{max} \times N \times F_s + t_{max} \times F_s \times X \times Z \times K$</td>
</tr>
<tr>
<td>Proposed Method</td>
<td>$t_{max} \times N \times F_s + Z \times X \times M$</td>
</tr>
</tbody>
</table>

instead of the wavefields over the network. Thus, our method shows obvious advantages in communication cost reduction. Here, we assume there are total $N$ sensors in $K$ clusters and every cluster has $M$ sensors (for simplicity, we assume the sensor number in every cluster is the same). As discussed above, to implement the real time imaging, instead of using the total time $T$, an analysis window $t_{max}$ is adopted and within the sliding window a normalization is applied, as Eq. (B.1). ($F_s$ denotes the sampling frequency.) The migration image is in the depth domain with the dimensionality as $Z(\text{depth}) \times X(\text{offset})$. The communication and computation cost analysis is shown in Table I. It is obvious that our method reduces the communication costs significantly, which is important for sensor network applications.

In terms of the computation cost, sink nodes in our design have the same imaging tasks with those in [14]. While in the following stacking (either summation or multiplication) step, our system deals with stacked images instead of wavefields, so the proposed approach also has a lighter imaging computation burden. Besides the imaging computation, WI computation is also required in our proposed method. Thanks to no velocity updates and the AOI selection, WI has the computation complexity as $O((Z \times X)^2)$. And, because the radix does not appear in the complexity, the proposed method has the computation complexity shown in Table I.

IV. SYNTHETIC EXPERIMENTS

In this section and Section V, we apply our proposed method and the centralized RTM method on synthetic and field seismic data to locate microseismic sources for validation. Because the centralized RTM has a higher resolution than the distributed RTM, the imaging accuracy comparison is made between the proposed approach and the centralized RTM. Synthetic examples use a portion of the Marmousi model [27] (shown in Fig. 4), which features a series of nearly parallel stratigraphic layers. The grid size is $300 \times 300$, with width and depth intervals are 4 m. The surface sensor array collected 300 traces. There are 9 clusters and each contains 60 traces.

A. Single Source Location

To avoid the interferences from different microseismic sources, we first simulate a single source situation. The source is located at (150, 750), shown in Fig. 4 as the yellow star. The time length of the recorded data is 0.5 s. Fig. 5 shows the energy map generated from the centralized RTM method, while Fig. 6 shows that from the proposed method. It is clear that the centralized RTM result has a finer resolution, which demonstrates the necessity of the WI.

In addition, although it seems accurate, the location results from both Fig. 5 and 6 are actually not. The maximum energy point on Fig. 5 is not (150, 750) but (149, 740), while the corresponding point on Fig. 6 is (149, 720), which is expected since the proposed method has a lower location resolution and accuracy than RTM due to the low stacking fold. We set a threshold as the 80% of the maximum energy, then an AOI map can be obtained, as shown in Fig. 7. Starting with the point (149, 720), a gradient descent based WI method is applied to achieve an optimal location result shown in Fig. 8. After the WI, the maximum energy point on Fig. 8 is (150, 750) which is the true source location.
B. Multiple Source Location

Next, we perform a multi-source location experiment on the same Marmousi model. The observation system retains the same parameters. There are 6 sources, and the locations of sources are shown in Fig. 4 using red and yellow stars. The time length of the recorded data is 1 s. Figs. 9 and 10 show the energy maps from the RTM and the proposed method, respectively. There are six clear bright spots, and the local maximum energy values are consistent with the true locations of the six sources. It shows that our algorithm can detect multiple sources simultaneously. Like the previous procedure used in the single source location experiment, we obtain the AOI with 80% of the maximum energy, shown in Fig. 11. Note that the AOI of a deeper source is larger than that of a shallower source, which is also true for the traditional RTM results in Figs. 5 and 9. Then, WI is applied to generate the final location result in Fig. 12, leading to a superior location resolution for both shallow and deep seismic sources.

C. Noise

To demonstrate noise resistant characteristics, we add strong noise to the recorded seismic data. Fig. 13 shows the recorded synthetic seismic data corresponding to the multiple source situation in Fig. 9, while Fig. 14 is the noisy data with 0 dB additive random noise. To show the details, we zoom in a small area with seismic sources and plot the data in the frames over the images. Fig. 15 demonstrates the energy map generated with the noisy data using the proposed distributed RTM method. It is almost the same as the result obtained from the noise free test shown in Fig. 10. In addition, the AOI extraction is also shown in the small frame, which is also similar to Fig. 11. Consequently, the final WI based location result is very close to the noise free situation.

To sum up, our proposed approach has a better resolution and location accuracy than the centralized RTM method [15], and lower communication and computation costs than the previous distributed RTM method [14]. In addition, our approach is also robust even when SNR is low.
Fig. 13: Recorded seismic data without noise.

Fig. 14: Recorded seismic data with 0 dB noise.

Fig. 15: Location result based on the noisy data using the proposed method. The AOI extraction result is also shown.

V. FIELD APPLICATION

Synthetic experiments with single and multiple sources have shown promising results based on the proposed method. In this section, we show a field application example. The field data were acquired by IRIS (Incorporated Research Institutions for Seismology) from June 21 to July 26, 2016 in Enid, Oklahoma, USA. We take one seismic line in the east-west direction in this study. The line includes 127 nodal sensors with the spacing around 100 m. The sampling rate is 250 Hz. During the one-month experiment, only two earthquakes occurred within 10 km range of the array. We choose a M2.3 earthquake on July 11, 2016 as the target and apply the RTM methods to locate the microseismic source.

We use a 1D velocity model from Oklahoma Geological Survey (OGS) for the source location. In addition, a relocation result has also been done in [28], where a modified velocity model was used. The earthquake source location results are shown in Figs. 17, 18 and 19. All figures show focused energies at a similar area, which indicates that the proposed method generates a reasonable result, and the velocity model from OGS is close to the real model.

Fig. 16: Field seismic data, which are filtered to $1 \sim 10$ Hz to highlight the signal and reduce high-frequency noise.

Fig. 17: Earthquake source location using the RTM method.

Fig. 18: Earthquake source location using the proposed distributed RTM method.

In Fig. 19, earthquake location results from OGS and [28] are marked as white and red stars, respectively. The source was located at \{lat 36.642439, lon -97.689520, depth 6560 m\} by OGS, \{lat 36.615857, lon -97.686024, depth 6051 m\} in [28], while our location result is \{lat 36.622840, lon -97.689920, depth 6325 m\}. The minor discrepancy between the focused point from our approach and previous earthquake location results can be caused by different methods and/or different data sets, as well as the velocity models.

1The complete waveform data for Enid experiment are available from IRIS Data Management Center (IRIS-DMC) at http://ds.iris.edu/mda/YW?timewindow=2016-2016 (last accessed January 8th, 2019).
We propose a joint imaging condition to address the computation of WI, which is close to the location results from OGS (white wavefields recorded at the geophones. The simulation of the seismic source, and in the backward propagation of the expansion of Eq. (A.1), which is used in the forward wavefield computation of RTM.

The RTM technique is derived by the Taylor's partial series local normalization operator with a sliding-window: two wavefield propagations represents the core and most heavy computation of RTM.

The 2-D seismic P wave propagation can be modeled as a scalar wave equation in a finite-differential form:

$$\frac{\partial^2 P(x, z, t)}{\partial x^2} + \frac{\partial^2 P(x, z, t)}{\partial z^2} - \frac{1}{c^2(x, z)} \frac{\partial^2 P(x, z, t)}{\partial t^2} = 0, \quad (A.1)$$

where $x$ and $z$ are spatial variables, $t$ is a time variable, $c$ is the medium velocity, and $P(x, z, t)$ is the pressure wavefield. The RTM technique is derived by the Taylor’s partial series expansion of Eq. (A.1), which is used in the forward wavefield propagation, representing the wave modeling emitted by a seismic source, and in the backward propagation of the wavefields recorded at the geophones. The simulation of the two wavefield propagations represents the core and most heavy computation of RTM.

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**Appendix A**

**Scalar Wave Equation**

The 2-D seismic P wave propagation can be modeled as a scalar wave equation in a finite-differential form:

$$\frac{\partial^2 P(x, z, t)}{\partial x^2} + \frac{\partial^2 P(x, z, t)}{\partial z^2} - \frac{1}{c^2(x, z)} \frac{\partial^2 P(x, z, t)}{\partial t^2} = 0, \quad (A.1)$$

where $x$ and $z$ are spatial variables, $t$ is a time variable, $c$ is the medium velocity, and $P(x, z, t)$ is the pressure wavefield. The RTM technique is derived by the Taylor’s partial series expansion of Eq. (A.1), which is used in the forward wavefield propagation, representing the wave modeling emitted by a seismic source, and in the backward propagation of the wavefields recorded at the geophones. The simulation of the two wavefield propagations represents the core and most heavy computation of RTM.

**Appendix B**

**Local Normalization**

The multiplication of wavefields destroys phase information and leads to an exponential growth of relative amplitudes. To deal with the issue of unbalanced amplitudes, we design a local normalization operator with a sliding-window:

$$I_N(x, t) = \frac{I(x, t)}{\max[I(x, t - \tau/2 : t + \tau/2)]}. \quad (B.1)$$

where $\tau$ is the window size. This operator normalizes a time slice of the wavefield by the maximum value in a local window centered around the time $t$.

**References**


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